SOLITONS, COLLAPSES AND TURBULENCE 2019

Ярославский государственный университет им. П.Г. Демидова
P.G. Demidov Yaroslavl State University

IX INTERNATIONAL CONFERENCE

SOLITONS, COLLAPSES AND TURBULENCE: ACHIEVEMENTS, DEVELOPMENTS AND PERSPECTIVES

(SCT-19) IN HONOR OF VLADIMIR ZAKHAROV'S 80TH BIRTHDAY

2019

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Landau Institute for Theoretical Physics
P.G. Demidov Yaroslavl State University
Lebedev Physical Institute
of the Russian Academy of Sciences
Novosibirsk State Univeristy
IT company “Tensor”

IX International Scientific Conference

“SOLITONS, COLLAPSES AND TURBULENCE: Achievements, Developments and Perspectives” (SCT-19) in honor of Vladimir Zakharov’s 80th birthday

ABSTRACTS

Yaroslavl, August 5–9, 2019

IX International Scientific Conference
SOLITONS, COLLAPSES AND TURBULENCE
Yaroslavl, August 5–9, 2019

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FARADAY WAVES AND DROPLETS IN QUASI-ONE-DIMENSIONAL BOSE GAS MIXTURE

F. Kh. Abdullaev[1,2], **A. Gammal**[3], **R. K. Kumar**[3], **L. Tomio**[2]

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Faraday waves in mixtures of Bose gases are studied by taking into account quantum fluctuations beyond the Gross-Pitaevskii mean-field formalism with the Lee-Huang-Yang term [1,2]. For that, a Bose-Einstein condensed binary atomic mixture is assumed trapped in cigar-type geometry, having the inter- and intra-species scattering lengths periodically varying in time [3,4]. The period of the Faraday patterns is shown to be quite sensitive to the estimated value obtained by the beyond mean-field contribution, which can be used to measure quantum fluctuations in the ground state of the quasi-one-dimensional mixture. By studying the influence of the above nonlinear periodic modulations on quantum droplet dynamics, we also show that nonlinear resonances are excited in the oscillation widths of the quantum droplets. Variational predictions confirm numerical simulations for the corresponding formalism.

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RECENT TRENDS IN NONLINEAR OPTICS AND PHOTONICS: A MATHEMATICAL MODELING PERSPECTIVE

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Nonlinear optics and photonics research remains an active field with new phenomena and applications to be explored in the immediate future. It also remains a platform to study similar processes in other fields including nonlinear waves. In this talk, I will point to some of the mathematical challenges and opportunities that one can identify to advance on the theoretical front.

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CONTACT STRUCTURE ON FANO VARIETY OF QUADRIC AND CONSERVATION LAWS OF HAMILTONIAN PDES

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Systems of n Hamiltonian PDEs of hydrodynamic type with Hamiltonian operator of first order admit $n + 2$ conservation laws. There is a geometrically motivated natural way to associate such systems to 1-ruled $n + 1$-dimensional surfaces in P^{n+3}. It turnes out that such PDEs are described in terms of contact structure on Fano variety of quadric.

GENERATION OF ROGUE WAVES FROM RANDOM WAVEFIELD OF HIGH NONLINEARITY

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We study numerically the integrable turbulence developing from random waves of high nonlinearity, in the framework of the focusing one-dimensional Nonlinear Schrodinger (NLS) equation,

$$i\psi_t + \psi_{xx} + |\psi|^2\psi = 0. \tag{1}$$

According to the previous studies [1, 2], this type of initial conditions represents one of the most promising candidate for the enhanced generation of rogue waves. We show that, after a short evolution, the integrable turbulence enters a *quasi-stationary state* (QSS) – the state in which its statistical characteristics change with time very slowly. The subsequent evolution toward the asymptotic *stationary* state turns out to be too long for our computational resources, and we focus instead on the examination of the basic statistical characteristics in the beginning of the QSS. We demonstrate that among these characteristics the wave-action spectrum and the PDF are almost universal functions, Fig. 1(a,b), independent of the spectrum and the level of nonlinearity of the initial conditions – even when initial conditions have rather generic non-symmetric spectra. In particular, in the QSS the wave-action spectrum is symmetric and continuous, and decays close to exponentially at large wavenumbers, while the PDF is strongly non-exponential (non-Rayleigh) with deviation from the exponential distribution,

$$\mathcal{P}_R(I) = e^{-I}, \tag{2}$$

by orders of magnitude at large intensities $I = |\psi|^2$. Moreover, we show that the PDF is very well approximated by a Bessel function,

$$\mathcal{P}_{QSS}(I) = 2\mathcal{K}_0(2\sqrt{I}), \tag{3}$$

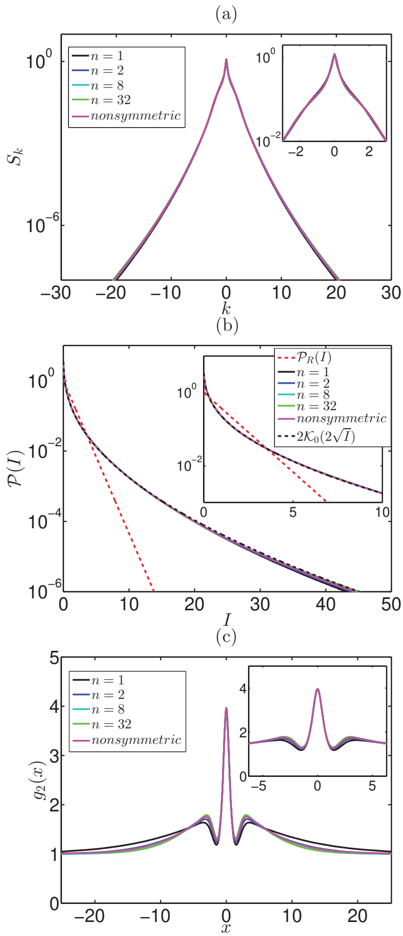

Fig. 1: *(Color on-line) Statistical characteristics in the beginning of the QSS for four super-Gaussian initial spectra* $\propto e^{-|k/\theta|^n}$ *with* $n = 1, 2, 8, 32$ *and one non-symmetric initial spectrum: (a) wave-action spectrum* S_k*, (b) PDF of intensity* $\mathcal{P}(I)$ *and (c) autocorrelation of intensity* $g^{(2)}(x)$*. The red dashed line shows the exponential PDF (2) and the black dashed curve indicates the Bessel fit (3). The insets in the figures show the same functions at larger scales.*

which is derived analytically under the assumption of large-scale spatial non-mixing of the wavefield, and we confirm this assumption by examining the evolution for individual realizations of incoherent wave input. The Bessel fit (3) for the PDF corresponds to the value of kurtosis $\kappa_4 = 4$, indicating a strong intermittency. In the QSS, the autocorrelation of intensity,

$$g^{(2)}(x) = \frac{\langle I(y+x)I(y)\rangle}{\langle I(y)\rangle^2}, \tag{4}$$

contains close to universal bell-shaped central part with the width of unity order, and also non-universal part, which depends on the spectrum and the nonlinearity level of the initial conditions, and corresponds to large-scale correlation.

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EINSTEIN EQUATIONS: SOLUTION-GENERATING METHODS AS "COORDINATE" TRANSFORMATIONS IN THE SOLUTION SPACE

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The complete integrability of certain two-dimensional symmetry reductions of vacuum Einstein equations was conjectured from the beginning of 1970th by R.Geroch [1], W.Kinnersley and D.Citre [2], D.Meison [3]. However, the actual discovery of integrability of these equations was made just forty years ago in a pioneer work of V.Belinski and V.Zakharov [4,5], who formulated the Inverse scattering approach to solution of these equations and presented the algorithm for generating of N-soliton solutions on any vacuum backgrounds with the same symmetry. Soon after that, the N-soliton solutions for Einstein - Maxwell equations generating on arbitrary electrovacuum backgrounds were found in the author's paper [6]. Later, different authors suggested some solution generating methods for vacuum Einstein equations and electrovacuum Einstein - Maxwell equations using different mathematical contexts – other formulations of the Inverse Scattering methods [7], B*ä*cklund or symmetry transformations, singular integral equation method as well as some others (see the book [8]). In the present paper, we describe various solution generating methods not by means of their action on some specially chosen particular solutions, but as transformations of ”coordinates” in the infinite dimensional space of solutions. Such ”coordinates” can be chosen, e.g. as the boundary values of the Ernst potentials on those degenerate orbits of two-dimensional space-time isometry group, where the behaviour of gravitational and electromagnetic fields is regular. In this case, the action of different solution generating methods is represented by simple explicit expressions which do not need any particular choice of the initial solution. The explicit form of these ”coordinate transformations” allow to find the interrelations between various solution generating methods as well as to determine the relations of the parameters of generating solutions with physical parameters of the constructing field configurations without/before performing detail calculations of the corresponding solutions.

Examples of new solutions for Einstein - Maxwell equations will be pesented also.

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EVOLUTION OF WEAKLY NONLINEAR RANDOM WAVE FIELDS: KINETIC EQUATIONS VS THE ZAKHAROV EQUATION

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We aim to understand the evolution of random water wave fields by using different statistical and dynamical models and compare the results with observations. The models we used include the kinetic equations (the standard Hasselmann equation (KE) and the generalized gKE [1,2]), the Zakharov equation, and the “mirror” Zakharov equation with the opposite sign of the kernel. The Zakharov equation is a weakly nonlinear truncation of the Euler equations, which does not invoke any statistical hypotheses, and the statistical description is obtained by averaging over an ensemble of realisations. The two kinetic equations are derived from the Zakharov equation using the standard statistical closure, the investigation of its role and validity being the primary target of the present study. The mirror Zakharov equation is an artificial tool which describes wave dynamics free from the modulational instability. Since the kinetic theory does not depend on the sign of the kernel, this equation leads to the same kinetic equations. We employ the novel algorithm for direct numerical simulation of the Zakharov equation (DNS-ZE), specifically aimed at statistical aspects of long-term wave field evolution.

Recently, it has been shown [3] that in the absence of wind the kinetic equations capture well the evolution of the bulk characteristics of a wave field (significant wave height, total energy, position of the spectral peak), but the shape of the spectra is considerably different from that obtained with the Zakharov equation. The DNS spectra are considerably broader and have lower spectral peaks. Simulations of the mirror Zakharov equation produced spectral evolution equivalent to the original equation, thus ruling out the modulational instability as a potential cause of the discrepancies. Comparison of simulations with KE, gKE and DNS-ZE with observation in real oceanic conditions shows that the DNS-ZE and observed spectra are indeed very close, while differing quite significantly from those simulated on the basis of the kinetic theory.

Here, we study evolution of both wave spectra and higher statistical moments of wave elevation by performing a detailed analysis of short-term evolution (about 10^2 characteristic periods) of random water wave fields without wind, choosing conditions for which detailed laboratory observations of the initial stages of both the spectral evolution and the evolution of higher statistical moments are available, as well as numerical simulations performed using other methods, such as higher-order spectral method and the Dysthe equation (Onorato et al 2009; Toffoli et al 2010; Xiao et al 2013). Since these results are based on laboratory measurements, the spectra under consideration have narrow spectral width (from nearly one-dimensional spectra to the angular width of a typical swell). The spectral evolution over the short initial stage is limited, and it is difficult to perform its meaningful comparison (although certain differences between spectra obtained by DNS and the kinetic theory are apparent even at the short term). Thus, the focus of this study is on the evolution of other statistical characteristics: the spectral variance between realisations, the first nontrivial higher statistical moment due to wave interactions (the dynamical kurtosis, which has important applications for the prediction of freak waves) and on the detailed behaviour of four-wave cumulants. For wave fields with small angular width, cumulants obtained with the original Zakharov equation are affected by the presence of modulational instability. The statistical closure is shown to underestimate higher statistical moments, giving results close to those obtained with the mirror Zakharov equation, with the opposite sign. For wave fields with large angular width higher statistical moments are shown to be small, justifying the neglect of the dynamical kurtosis.

The results of simulations provide a novel insight into the nature of evolution of random wave fields and the degree of validity of commonly adopted statistical description based upon the standard closure. The established evolution of kurtosis sheds new light on freak waves and, in particular, on the shortcomings of their description employing the standard closure.

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THE GENERALIZED PHILLIPS SPECTRUM AND WIND-WAVE DISSIPATION

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We consider an extension of the kinetic equation by [3] that takes into account effects of four-wave resonant interactions and dissipation due to intermittent wave breaking. The validity of this asymptotic model in the whole range of wave scales is supported by nonlinear dissipation function that depends on dimensionless spectral flux. Under additional physical assumptions "a local substitute" of this function can be proposed in a form widely used in the today spectral wave models

$$S_{diss}(\mathbf{k}) = \beta\omega \left(B(\mathbf{k})\right)^r E(\mathbf{k})\theta(\omega - \omega_c) \qquad (1)$$

Here $E(\mathbf{k})$ – energy wavenumber spectrum and $B(\mathbf{k}) = E(\mathbf{k})|\mathbf{k}|^4$ ($\mathbf{k}$ being wavevector, θ – the Heaviside function) is the Phillips dimensionless saturation function [1]. In stationary case this dissipation function describes a transition from the direct cascade Komogorov-Zakharov (KZ) spectrum $E \sim \omega^{-4}$ [5] to the Phillips spectrum $E \sim \omega^{-5}$ [4]. The Heaviside function θ in (1) ensures a low frequency cutoff of the dissipation function. The nonlinear dissipation (1) determines, first, the exponents of the asymptotic solutions and, secondly, observability of the transition between two regimes of KZ and Phillips' spectra.

Based on experimental estimates of the transition frequency range [2] the coefficient β and exponent r of the dissipation function (1) can be evaluated. This issue is detailed for results of numerical solutions of the kinetic equation for duration- and fetch-limited setups with exact collision integral S_{nl}.

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POINT EQUIVALENCE OF SECOND-ORDER ODES TO THE PAINLEVÉ EQUATIONS

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Problem of equivalence of the second-order equations

$$\frac{d^2y}{dx^2} = S(x,y)\Big(\frac{dy}{dx}\Big)^3 + 3R(x,y)\Big(\frac{dy}{dx}\Big)^2 + 3Q(x,y)\frac{dy}{dx} + P(x,y) \quad (1)$$

with respect to point change of variables

$$z = \xi(x,y), \qquad w = \eta(x,y), \qquad \det\frac{\partial(\xi,\eta)}{\partial(x,y)} \neq 0 \quad (2)$$

can be solved with the use of invariants of the equivalence transformation group of this family of equations, generated by transformations (2). If two equations of the form (1) are related by a change of variables (2), then all invariants of these equations are equal. The problem of constructing invariants of the family (1) was under study many times, beginning with R. Liouville and A. Tresse. Here the corresponding result from [1] is used. The Painlevé equations PI–PVI, and equation P34 as well, related to PII by a Miura type substitution, fall into degenerate types of equations (1). For them, the basis of algebraic invariants consists of two invariants

$$I_1(x,y), \qquad I_2(x,y). \tag{3}$$

Arbitrary invariant can be obtained from the basis ones by applying the invariant differentiations D_1, D_2 and algebraic operations.

Necessary conditions of the equivalence to a Painlevé equation (written in variables z, w) are found in the following way. One should calculate the basis invariants $I_1(z,w)$, $I_2(z,w)$ of this equation and a number of derivative invariants $D_j I_k$, $D_i D_j I_k$, i, j, $k = 1, 2$. A part of the invariants is used for eliminating z, w and constant parameters of the Painlevé equation. This allows to obtain the formulas of the change of variables (2), where the right-hand side is given by a function of invariants (3) of equation (1) and their derivatives. As a result the relations remain, which connect the invariants only and represent the necessary equivalence conditions. To prove the sufficiency, these conditions are applied to an equation

$$\frac{d^2y}{dx^2} = f(x,y). \tag{4}$$

Let this lead to a canonical form of the corresponding Painlevé equation, up to the transformations preservind the form of equations (4). This means the sufficiency of the equivalence conditions applied to (4). Canonical forms of the Painlevé eqiations defined by ODE with right-hand side, that does not depend on the first derivative, have been found in [2] (PI and PII are just in the canonical form). In constructing the equivalence criteria it is sufficient to use the invariants I_1, I_2, D_2I_2, $D_2^2I_2$ for PI, invariants I_1, I_2, D_jI_2, $D_jD_2I_2$, $j = 1, 2$ for PII, invariants I_1, I_2, D_1I_2, D_2I_1, D_1I_1 for P34, invariants I_1, I_2, D_1I_2, D_2I_1, D_1I_1, $D_1^2I_1$ for PIV, invariants I_1, I_2, D_2I_1, D_1I_1, $D_1^2I_1$, $\ldots$, $D_1^mI_2$ for PIII ($m = 3$), PV ($m = 4$) and PVI ($m = 5$). Instead of the invariants $D_1^mI_1$ we use the series of invariants J_{m+1} introduced in [3] which is the modification of known Liouville series of invariants j_{2m}.

Conditions of the equivalence to the first Painlevé equation with respect to the point transformations (2) have been obained in [4] and later in [5]. Some necessary equivalence conditions for PI–PIV are constructed in [6] in terms if the invariants j_{2m}. Non-complete necessary conditions of the equivalence to PII, PIV, P34 are found in [5,7,8]. In [1] the complete set of necessary conditions of the equivalence to PI, PII, PIV and P34 were obtained. Then in [9] some conditions were added to necessary equivalence conditions for PII and P34 found earlier in [5,8]. But their sufficiency has not been proved.

Today the criteria of equivalence to the PI–PIV and P34 equations are proved [3,10,11]. Necessary conditions of the equivalence to PV equation are stated as a system of relations in invariants J_m, variable w and an auxiliary variable H [12]. In theory, variables H and w can be eliminated, but this leads to cumbersome relations useless to solve the problem of equivalence of an ODE (1) to PV. Instead of this we propose the procedure of eliminating H and factorizing the remaining overdetermined system, that allows to find the expression for w in the change of variables (2). Substitution of such found H and w should turn the relations of the system into identities. Necessary conditions of the equivalence to PVI equation are stated as a system of relations in invariants J_m and variables z, w and H [13]. We apply similar procedure of eliminating H and finding the expressions for z, w in the change of variables (2).

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RAMAN COMPRESSION OF LASER PULSES IN WEDGE-SHAPED JET PLASMA

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The features of Raman backscattering of laser pulses in a wedge-shaped jet plasma with a significant density gradient are analyzed. The possibility of compensating for an excessively large pump chirp by using of density inhomogeneity along the gas jet is shown. In this case, Raman compression occurs without a significant loss of energy efficiency. The dependence of efficiency on the magnitude of the pumping chirp has an asymmetrical form related to the preference of using a denser plasma at the initial stage. The conditions for optimal compression are determined.

Moreover, the possibility of obtaining a high-energy output signal using wide-aperture laser pulses in a wedge-shaped plasma is shown. At this, Raman amplification keeps high the efficiency and the focusability as an usual rectangular plasma. To compensate density gradient in wedge-shaped plasma one may use transverse chirping of pump pulse or very short seed pulse. In last case reasonable limitations on seed pulse duration and jet parameters will be applied. For example, the seed duration should be less 40 fs and plasma length variation should be less than 40% of the average amplification length. The main limiting factor for good focusing of the output pulse in this scheme are aberrations during the passage of a heterogeneous jet plasma. The parameters of the gas jet for which the aberrations can be neglected are determined.

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COHERENT PROPAGATION AND COMPRESSION OF LASER PULSES IN OPTICAL MULTI-CORE FIBER

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The propagation of laser pulses in MCF from the central core and an even number of cores located around the ring is studied. Approximate quasi-soliton homogeneous solutions of the wave field in the considered MCF are found. The stability of the in-phase soliton distribution is analytically and numerically shown. At low energies, its wave field is distributed over all MCF cores and has a duration many times (5-6 times) greater than the duration of the NSE soliton with the same energy. Contrary, almost all of the radiation at high energies is located in the central core with a duration similar to NSE soliton. The transition between two types of distributions is very sharp and occurs at critical energy, which is weakly dependent on the number of cores and on the coupling coefficient with the central core. The self-compression mechanism of laser pulses was proposed. It consist in injection into such MCF a wave packet close to the determined soliton with an energy larger than the critical value. It is shown that the degree of compression weakly depends on the energy and the number of cores and is approximately equal to 6 times with almost 100% energy efficiency. The use of longer laser pulses allows one to increase the degree of compression of more than 30 times with an energy efficiency of more than 50%. The obtained analytical estimates of the degree and efficiency of the compression are in good agreement with the results of numerical simulation.

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SIX-DIMENSIONAL HEAVENLY EQUATION AND RELATED SYSTEMS. DRESSING SCHEME AND THE HIERARCHY

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We consider self-dual Yang-Mills equations for the Lie algebra of vector fields as a natural reduction in the framework of a general linearly degenerate dispersionless hierarchy. We define the reduction in terms of wave functions and introduce generating relations, Lax-Sato equations and the dressing scheme for the reduced hierarchy. In particular, six-dimensional heavenly equation and related systems are discussed.

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INVESTIGATION OF SOLITON COMMUNICATION LINES

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Modern optical fiber communication lines have capacity limit associated with dispersive and nonlinear effects. The signal envelope that carries information is described by nonlinear Schrodinger equation (NSE). NSE has solution in the form solitary wave – soliton, in the focusing case. The envelope of soliton does not change in the line due to balance dispersive expansion and nonlinear compression. This fact attracts the interest in investigation soliton communication systems.

In this research the capacity of soliton communication lines is studied. The numerical modeling of soliton signals propagation in Gauss channel is performed. Data transmission in 16 frequency channels considering dispersion and nonlinearity is made. In order to increase the spectral efficiency soliton phases and shifts for coding information are used. The problem of soliton interaction compensation and compression in one frequency channel is considered.

Thus, the possibility of data transmission for distance 2000 km with capacity 100 Gbits/s and spectral efficiency 2 bits/s/Hz is demonstrated.

POLYNOMIAL INTEGRABLE HAMILTONIAN SYSTEMS AND SYMMETRIC POWERS OF $\mathbb{C}^2$

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We give a construction of polynomial integrable systems in $\mathbb{C}^{2N}$ (or on $\mathbb{R}^{2N}$, if the base field is $\mathbb{R}$) using the algebra-geometric structure of the space $\mathrm{Sym}^{\mathrm{N}}(\mathbb{C}^2)$. It is based on a canonical transformation $\varphi : \mathbb{C}^{2N} \to \mathbb{C}^{2N}$ from variables $(\mathbf{x}, \mathbf{y}) \in \mathbb{C}^{2N}$, $\mathbf{x} = (x_1, \ldots, x_N)$, $\mathbf{y} = (y_1, \ldots, y_N)$ to $\mathbf{q} = (q_1, \ldots, q_N)$, $\mathbf{p} = p_1, \ldots, p_N)$ given by the generating function

$$G = \sum_{i,n=1}^{N} \frac{1}{n} x_i^n p_n \;\Rightarrow\quad q_n = \frac{\partial G}{\partial p_n} = \frac{1}{n} \sum_{i=1}^{N} x_i^n, \quad y_i = \frac{\partial G}{\partial x_i} = \sum_{n=1}^{N} x_i^{n-1} p_n \,.$$

The canonical transformation φ can be decomposed in the standard projection $\pi : \mathbb{C}^{2N} \to (\mathbb{C}^2)^N / S_N = \mathrm{Sym}^{\mathrm{N}}(\mathbb{C}^2)$ and the explicit birational isomorphism $\mathrm{Sym}^{\mathrm{N}}(\mathbb{C}^2) \to \mathbb{C}^{2\mathrm{N}}$. The projection π gives a branching covering of $\mathrm{Sym}^{\mathrm{N}}(\mathbb{C}^2)$.

With any polynomial $F(x,y) \in \mathbb{C}[x,y]$ such that $\partial_y F(x,y) \neq 0$ we associate N compatible Stäkel type integrable Hamiltonian systems in $\mathbb{C}^{2N}$

$$\frac{d\,x_i}{d\,t_k} = \frac{\partial\, H_k(\mathbf{x},\mathbf{y})}{\partial\, y_i}, \quad \frac{d\,y_i}{d\,t_k} = -\frac{\partial\, H_k(\mathbf{x},\mathbf{y})}{\partial\, x_i}, \qquad i,k \in \{1,\ldots,N\},$$

where $H_k(\mathbf{x},\mathbf{y}) = \sum_{i=1}^{N} W_{k,i} F(x_i,y_i)$ and $W = (W_{k,i})$ is the inverse Vandermonde matrix. The intersection of the level sets $H_s(\mathbf{x},\mathbf{y}) = h_s,\ h_s \in \mathbb{C},\ s = 1,\ldots,N$, is a quasi-projective algebraic variety in $\mathbb{C}^{2N}$

$$\mathcal{G} = \{(\mathbf{x},\mathbf{y}) \in \mathbb{C}^{2N} \,|\, x_i \neq x_j \text{ if } i \neq j, \text{and } F(x_i,y_i) = \sum_{s=1}^{N} h_s x_i^{s-1},\ i = 1,\ldots,N\}.$$

which is S_N invariant with the free action of S_N.

We show that the functions $\mathcal{H}_k(\mathbf{q},\mathbf{p}),\ k = 1,\ldots,N$, defined by $\phi^*\mathcal{H}_k(\mathbf{q},\mathbf{p}) = H_k(\mathbf{x},\mathbf{y})$ are polynomials. They are functionally independent. It leads us to one of our main result:
In the space $\mathbb{C}^{2N}$ there are N commuting *polynomial* Hamiltonian systems correponding to the Hamiltonians $\mathcal{H}_1(\mathbf{q},\mathbf{p}),\ldots,\mathcal{H}_N(\mathbf{q},\mathbf{p})$.

It follows from the Liouville theorem that all Hamiltonian systems obtained are completely integrable. In the results obtained we do not impose any condition on the genus of the curve

$$\Gamma = \{(x,y) \in \mathbb{C}^2 \mid F(x,y) = \sum_{s=1}^{N} h_s x^{s-1}\}$$

neither request that the curve Γ is regular.

In the case of a non-singular hyperelliptic curves Γ of genus g and $N = g$ our systems represent integrable hierarchies of equations which had been discovered in the theory of finite gap solutions (algebra-geometric integration) of the Korteweg-de-Vrise equation. Then there is defined g-dimensional sigma function $\sigma(\mathbf{u})$, where $\mathbf{u} = (u_1,\ldots,u_g)$, and Abelian functions $\wp_{i,j}(\mathbf{u}) = -\partial_{u_i}\partial_{u_j} \ln\sigma(\mathbf{u})$. Using the properties of the hyperelliptic functions $\wp_{i,j}(\mathbf{u})$, we can present the simultaneous solutions of the Hamiltonian systems obtained in the form $(\mathbf{q}(\mathbf{u}),\mathbf{p}(\mathbf{u}))$, where the entries of the vector $\mathbf{q}(\mathbf{u})$ are polynomial functions in $\wp_{g,i}(\mathbf{u})$, and the entries of the vector $\mathbf{p}(\mathbf{u})$ are linear functions in $\wp_{g,g,i}(\mathbf{u})$, whose coefficients are polynomials in $\wp_{g,i}(\mathbf{u}),\ i = 1,\ldots,g$.

For $N = 2, 3$ and $g = 1, 2, 3$ we present the examples of our polynomial systems and give an explicit description of the solutions of these systems.

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GENERALIZATION OF THE GIVENTAL THEORY FOR THE ORIENTED WDVV EQUATIONS

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The WDVV equations, also called the associativity equations, is a system of non-linear partial differential equations for one function, that describes the local structure of a Frobenius manifold. In enumerative geometry the WDVV equations control the Gromov-Witten invariants in genus zero. In his fundamental works, A. Givental interpreted solutions of the WDVV equations as cones in a certain infinite-dimensional vector space. This allowed him to introduce a group action on solutions of the WDVV equations, which proved to be a powerful tool in the study of these solutions and, in particular, in Gromov-Witten theory. I will talk about a generalization of the Givental theory for the oriented WDVV equations and an application to the open Gromov-Witten invariants.

POLYNOMIAL GRAPH INVARIANTS AND LINEAR HIERARCHIES

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Let $W_G(q_1, q_2, \ldots)$ be a weighted symmetric chromatic polynomial of a graph G. S. Chmutov, M. Kazarian and S. Lando in the paper arXiv:1803.09800v2 proved that the generating function $\mathcal{W}(G)$

for the polynomials $W_G(q_1, q_2, \ldots)$ is a τ-function of the Kadomtsev–Petviashvili integrable hierarchy. I will prove that the function $\mathcal{W}(G)$ itself is a solution of a linear integrable hierarchy. Also I describe the initial conditions for the general formal τ-function of the KP-hierarchy which guarantee that the τ-function is a solution of a linear integrable hierarchy. My talk based on the joint work with A. Mikhailov.

ALGEBRAICALLY SOLVABLE SYSTEMS OF NONLINEAR ORDINARY DIFFERENTIAL EQUATIONS

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A review will be presented of recent approaches to identify algebraically solvable systems of nonlinear ordinary differential equations (ODEs); with particular attention to systems of 2 first-order ODEs with polynomial right-hand sides:

$$\dot{x}_n = P^{(n)}(x_1, x_2) \ , \quad n = 1, 2 \ ,$$

with $P^{(n)}(x_1, x_2)$ explicit polynomials of low degree in x_1 and x_2. Some of the results which shall be reported have been obtained with **Robert Conte** and **François Leyvraz,** and with **Farrin Payandeh**.

SOLITONS AND BREATHERS: AN ENVIRONMENTAL FLUID MECHANICS PERSPECTIVE

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Integrable nonlinear wave equations such as the nonlinear Schrödinger or Korteweg-de Vries equation have proven their usefulness and efficiency in wave modelling throughout different fields of physics and engineering. Exact solutions of both evolution equations are known as soliton as well as breathers and can shape nonlinear waves to propagate in either a stationary or pulsating manner. These localized structures have been for instance observed in optics, plasma, Bose-Einstein condensates and hydrodynamics. The applicability of these to real-world problems will be elaborated, especially, from an environmental fluid mechanics vantage point. This includes for instance the modelling and prediction of rogue and tsunami waves.

NONLINEAR DISPERSION WITH QUANTUM OSCILLATOR

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Using the Gauss-Hermite functions, one can investigate the nonlinear dispersion managed soliton with quantum oscillator. We obtain an infinite dimensional discrete dynamical system. It is shown it has a similar structure with the non-linear Schrodinger equation but the conserved densities are different. Then we can study the stationary solution. Also, the pseudo-potential method is used to prove one soliton solution exists.

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ROGUE WAVES ON THE PERIODIC AND DOUBLE-PERIODIC BACKGROUND

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We address Lax–Novikov equations derived from the cubic NLS equation. Lax-Novikov equations of the lowest orders admit explicit periodic and double-periodic solutions expressed as rational functions of Jacobian elliptic functions. By applying an algebraic method which relates the periodic potentials and the squared periodic eigenfunctions of the Lax operators, we characterize *explicitly* the location of eigenvalues in the periodic spectral problem away from the imaginary axis. We show that Darboux transformations with the periodic eigenfunctions remain in the class of the same periodic waves of the NLS equation. On the other hand, Darboux transformations with the non-periodic solutions to the Lax equations produce rogue waves on the periodic background which are formed in a finite region of the time-space plane. The results are based on the recent papers [1,2,3].

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PECULIARITIES OF THE OSCILLATING WAVE PACKETS (BREATHERS) INTERACTION WITHIN THE FRAMEWORK OF THE MODIFIED KORTEWEG - DE VRIES EQUATION

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The present work is dedicated to the study of oscillating wave packets (breathers) which can exist in various nonlinear media. The peculiarities of interaction of breathers with each other and breathers with solitons are considered within the framework of the modified Korteweg - de Vries equation. The conditions when wave interaction leads to a significant increase in the amplitude of the resulting pulse are determined. Numerical simulations of breather turbulence dynamics are performed. Different statistical properties of breather turbulence (statistical moments, distribution functions) are analyzed. For a better understanding of complex multi-wave dynamics, the interaction modes of pair breather collisions are studied; possible shapes and properties of the resulting pulses are analyzed in a wide range of wave parameters.

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SOLITON INTERACTIONS IN THE SYSTEM OF SUPERCOMPACT EQUATIONS FOR COUNTER PROPAGATING 1D WAVES

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We apply a canonical transformation to a water wave equations to remove cubic nonlinear terms and to drastically simplify fourth-order terms in the Hamiltonian. This transformation from natural Hamiltonian variables η, ψ to new complex normal variables c, c^* explicitly uses the fact of vanishing exact four-wave interaction for water gravity waves for a 2D potential fluid. The new variable $c(x,t) = c^+(x,t) + c^-(x,t)$ is the sum of two analytic functions: $c^+(x,t)$ is analytic in the upper half-plane, $c^-(x,t)$ is analytic in the lower-plane. We obtain the system of two coupled differential equations for c^+ and c^- which is very suitable for analytical studies and numerical simulations:

$$\frac{\partial c^+}{\partial t} + i\hat{\omega}c^+ = \partial_x^+ \left[i(|c^+|^2 - |c^-|^2)c_x^+ + c^+\hat{k}(|c^+|^2 - |c^-|^2) - ic^+c^-c_x^{-*} - c^{-*}\hat{k}\left(c^+c^-\right) \right],$$
$$\frac{\partial c^-}{\partial t} + i\hat{\omega}c^- = \partial_x^- \left[i(|c^-|^2 - |c^+|^2)c_x^- - c^-\hat{k}(|c^-|^2 - |c^+|^2) - ic^-c^+c_x^{+*} + c^{+*}\hat{k}\left(c^+c^-\right) \right]. \quad (1)$$

Here $\hat{\omega}$ and $\hat{k}$ correspond to the multiplication by $\sqrt{gk}$ and $|k|$ in the Fourier space, $*$ denotes complex conjugation, the subscript $_x$ is the derivative with respect to the variable x, the differentiation operators ∂_x^+ and ∂_x^- are $ik\theta(k)$ and $ik\theta(-k)$, where $\theta(k)$ is the Heaviside step function.

We perform numerical simulation of the head-on breather collisions by using the system of supercompact equations (1). To generate initial conditions we use the Petviashvili method for finding breathers propagating in opposite directions. We perform detailed study of breather collision dynamics depending on their relative phase.

The research of the dynamics of breather interactions in the system of supercompact equations was supported by the Russian Science Foundation (Grant No. 18-71-00079). Work of A.I. Dyachenko (derivation of the system of supercompact equations for counter propagation 1D waves) was supported by state assignment Dynamics of the complex materials.

ON GENERALIZATION OF TAYLOR-GREEN VORTEX SOLUTION OF THE NAVIER-STOKES SYSTEM OF EQUATIONS

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Classical Taylor-Green solution (1937) of the Navier-Stokes system of equations

$$\vec{U}_t + (\vec{U}\cdot\vec{\nabla})\vec{U} - \mu\Delta\vec{U} + \vec{\nabla}P = 0, \quad (\vec{\nabla}\cdot\vec{U}) = 0 \tag{1}$$

where

$$\vec{U}(\vec{x},t) = cos(x)sin(y), \quad V(\vec{x},t) = -sin(x)cos(y)\exp(-2\mu t),$$

$$W(\vec{x},t) = 0, \quad P(\vec{x},t) = -\frac{1}{4}(\cos(2x)+\cos(2y))exp(-4\mu t),$$

describes properties of the vortex-flows of an incompressible fluid in two-dimensional non stationary case.

In a presented Report will be told about a more general case of vortex solutions of the system (1) starting from the stationary velocity of the flow having the form

$$\vec{U}(\vec{x},t) = A\cos(ax)\sin(by)\sin(cz), \;\; V(\vec{x},t) = B\sin(ax)\cos(by)\sin(cz),$$

$$W(\vec{x},t) = C\sin(ax)\sin(bx)\cos(cz). \tag{2}$$

Theorem 1 *Solutions of the Navier-Stokes system of equations with the components of velocity*

$$U(x,y,z,t) = a - A\cos(a\,(x-at))\sin(\frac{aB\,(y-bt)}{A})\sin(\frac{(Aa+Bb)\,(z-ct)}{C}),$$

$$V(x,y,z,t) = b - B\sin(a\,(x-at))\cos(\frac{aB\,(y-bt)}{A})\sin(\frac{(Aa+Bb)\,(z-ct)}{C}),$$

$$W(x,y,z,t) = c - (Aa+Bb)\,A\sin(a\,(x-at))\sin(\frac{aB\,(y-bt)}{A})\times$$

$$\times\cos(\frac{(Aa+Bb)\,(z-ct)}{C})C^{-1}a^{-1}$$

and with the function of the pressure of the form

$$4\,P(x,y,z,t)Aa^2C^2 = -2\,a^2A^3C^2\left(\cos(a\,(x-at))\right)^2 - 2\,a^2B^2\left(\cos(\frac{aB\,(y-bt)}{A})\right)^2 C^2A+$$

$$+2\,a^2A^5\left(\cos(\frac{(Aa+Bb)\,(z-ct)}{C})\right)^2\left(\cos(a\,(x-at))\right)^2 - 4\,A^4\left(\cos(\frac{(Aa+Bb)\,(z-ct)}{C})\right)^2 Bba+$$

$$+2\,a^2A^5\left(\cos(\frac{(Aa+Bb)\,(z-ct)}{C})\right)^2\left(\cos(\frac{aB\,(y-bt)}{A})\right)^2 - 2\,A^3\left(\cos(\frac{(Aa+Bb)\,(z-ct)}{C})\right)^2 B^2b^2+$$

$$+A^3a^2C^2\cos(2\,\frac{(Aa+Bb)\,(z-ct)}{C}) + 2\,a^2B^2\left(\cos(\frac{aB\,(y-bt)}{A})\right)^2 C^2A\left(\cos(a\,(x-at))\right)^2 -$$

$$-2\,a^2A^5\left(\cos(\frac{(Aa+Bb)\,(z-ct)}{C})\right)^2 + 2\,a^2A^3\left(\cos(\frac{aB\,(y-bt)}{A})\right)^2 C^2\left(\cos(a\,(x-at))\right)^2 +$$

$$+4\,a^3\mu\,\sin(\frac{aB\,(y-bt)}{A})\sin(\frac{(Aa+Bb)\,(z-ct)}{C})B^2C^2\sin(a\,(x-at))+$$

$$+4\,A^4\left(\cos(\frac{(Aa+Bb)\,(z-ct)}{C})\right)^2 Bb\left(\cos(\frac{aB\,(y-bt)}{A})\right)^2 a+$$

$$+2\,a^2B^2\left(\cos(\frac{aB\,(y-bt)}{A})\right)^2\left(\cos(\frac{(Aa+Bb)\,(z-ct)}{C})\right)^2 C^2A+$$

$$+4\,a^3A^4\mu\,\sin(\frac{aB\,(y-bt)}{A})\sin(\frac{(Aa+Bb)\,(z-ct)}{C})\sin(a\,(x-at))+$$

$$+4\,A^2\mu\,\sin(\frac{aB\,(y-bt)}{A})\sin(\frac{(Aa+Bb)\,(z-ct)}{C})B^2b^2a\sin(a\,(x-at))+$$

$$+8\,a^2A^3\mu\,\sin(\frac{aB\,(y-bt)}{A})\sin(\frac{(Aa+Bb)\,(z-ct)}{C})Bb\sin(a\,(x-at))-$$

$$-2\,a^2B^2\left(\cos(\frac{aB\,(y-bt)}{A})\right)^2\left(\cos(\frac{(Aa+Bb)\,(z-ct)}{C})\right)^2 C^2A\left(\cos(a\,(x-at))\right)^2 -$$

$$-2\,a^2A^3\left(\cos(\frac{(Aa+Bb)\,(z-ct)}{C})\right)^2 C^2 + 2\,A^3\left(\cos(\frac{(Aa+Bb)\,(z-ct)}{C})\right)^2 B^2b^2\left(\cos(\frac{aB\,(y-bt)}{A})\right)^2 -$$

$$-2\,a^2A^3\left(\cos(\frac{aB\,(y-bt)}{A})\right)^2\left(\cos(\frac{(Aa+Bb)\,(z-ct)}{C})\right)^2 C^2\left(\cos(a\,(x-at))\right)^2 -$$

$$-2\,A^3\left(\cos(\frac{(Aa+Bb)\,(z-ct)}{C})\right)^2 B^2b^2\left(\cos(\frac{aB\,(y-bt)}{A})\right)^2\left(\cos(a\,(x-at))\right)^2 -$$

$$-4\,A^4\left(\cos(\frac{(Aa+Bb)\,(z-ct)}{C})\right)^2 Bb\left(\cos(\frac{aB\,(y-bt)}{A})\right)^2 a\left(\cos(a\,(x-at))\right)^2 -$$

$$-2\,a^2A^5\left(\cos(\frac{(Aa+Bb)\,(z-ct)}{C})\right)^2\left(\cos(\frac{aB\,(y-bt)}{A})\right)^2\left(\cos(a\,(x-at))\right)^2 +$$

$$+4\,a^3A^2\mu\,\sin(\frac{aB\,(y-bt)}{A})\sin(\frac{(Aa+Bb)\,(z-ct)}{C})C^2\sin(a\,(x-at))+$$

$$+4\,A^4\left(\cos(\frac{(Aa+Bb)\,(z-ct)}{C})\right)^2 Bba\left(\cos(a\,(x-at))\right)^2 +$$

$$+2\,a^2A^3\left(\cos(\frac{(Aa+Bb)\,(z-ct)}{C})\right)^2 C^2\left(\cos(a\,(x-at))\right)^2 +$$

$$+2\,A^3\left(\cos(\frac{(Aa+Bb)\,(z-ct)}{C})\right)^2 B^2b^2\left(\cos(a\,(x-at))\right)^2 + 4\,Aa^2C^2C_1 + 2\,a^2A^3C^2$$

are hold when between of the parameters of the flow exists the relation

$$A^4a^2 + 2\,A^3aBb + \left(a^2C^2 + B^2b^2\right)A^2 + B^2a^2C^2 = 0.$$

As example for the values of parameters $a = 0.153$, $b = 309$, $B = 300\ldots444$, $C = 3\ldots10$ there are exist values of A:

$$A = RootOf\left(\_Z^4a^2 + 2\,\_Z^3aBb + \left(a^2C^2 + B^2b^2\right)\_Z^2 + B^2a^2C^2\right).$$

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ROGUE WAVES AND ANALOGIES IN OPTICS AND OCEANOGRAPHY

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We review the study of rogue waves and related instabilities in optical and oceanic environments, with particular focus on recent experimental developments. In optics, we emphasize results arising from the use of real-time measurement techniques, whilst in oceanography we consider insights obtained from analysis of real-world ocean wave data and controlled experiments in wave tanks. Although significant progress in understanding rogue waves has been made based on an analogy between wave dynamics in optics and hydrodynamics, these

comparisons have predominantly focused on one-dimensional nonlinear propagation scenarios. As a result, there remains significant debate about the dominant physical mechanisms driving the generation of ocean rogue waves in the complex environment of the open sea. Here, we review state-of-the-art of rogue wave studies in optics and hydrodynamics, aiming to clearly identify similarities and differences between the results obtained in the two fields. In hydrodynamics, we take care to review results that support both nonlinear and linear interpretations of ocean rogue wave formation, and in optics, we also summarise results from an emerging area of research applying the measurement techniques developed for the study of rogue waves to dissipative soliton systems. We conclude with a discussion of important future research directions [1].

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NEW INTEGRALS OF MOTION FOR WATER WAVES

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1-D waves on the free surface of deep water can be separated on two subsets: waves moving to the right and wave moving to the left. System of two equation describing evolution of these two subsets is derived. It became possible due to specific feature of four-wave interaction of water waves. An important consequence of this property was the conservation of of ”numbers of waves” moving to the right and to the left *separately*. Also the system of envelope equations for two counter-streaming waves is obtained[1].

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DISCRETE PAINLEVÉ EQUATIONS IN TILING PROBLEMS

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The notion of a *gap probability* is one of the main characteristics of a probabilistic model. In [1] Borodin, extending to the discrete case a well-known relationswhip between gap probabilities and differential Painlevé equations, showed that for some discrete probabilistic models of Random Matrix Type discrete gap probabilities can be expressed through solutions of discrete Painlevé equations, which provides an effective way to compute them [2]. We discuss this correspondence for a particular class of models of lozenge tilings of a hexagon. For uniform probability distribution, this is one of the most studied models of random surfaces. Borodin, Gorin, and Rains [3] showed that it is possible to assign a very general elliptic weight to the distribution and degenerations of this weight correspond to the degeneration cascade of discrete polynomial ensembles, such as Racah and Hahn ensembles and their q-analogues. This also correspond to the degeneration scheme of discrete Painlevé equations, due to the work of Sakai. Continuing the approach of Knizel [4], we consider the q-Hahn and q-Racah ensembles and corresponding discrete Painlevé equations of types $q-P(A_2^{(1)})$ and $q-P(A_1^{(1)})$ [5]. We show how to use the algebro-geometric techniques of Sakai's theory to pass from the isomonodromic coordinates of the model to the discrete Painlevé coordinates that is compatible with the degeneration.

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ABOUT VORTEX TURBULENCE FORMATION BY A HEAT FLUX IN SUPERFLUID HELIUM IN A LONG CAPILLARY

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An experimental study of the propagation of long-term (up to 1000 μs) heat pulses in superfluid helium showed that the formation of a vortex field near the heater at characteristic dimensions of the order of several millimeters is crucial for heat transfer. This vortex congestion blocks further counterflow of the normal and superfluid components in volume of waveguide and development of turbulence in the entire waveguide at the all capillary length. These results explain the weak dependence of the testing single pulse reduction at propagation along the capillary with constant heat flux, which we observed at experiments. The constant heat flux would have to produce a spatially homogeneous vortex density in all volume of the capillary and exponential decreasing of the amplitude of the testing heat pulse at the waveguide distance, while in experiments the dependence was close to distance independent at 40 times increase in length of waveguide.

WAVE-MEAN FLOW INTERACTIONS IN DISPERSIVE HYDRODYNAMICS

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The interaction of waves with a mean flow is a fundamental and longstanding problem in fluid mechanics. The key to the study of such an interaction is the scale separation, whereby the length and time scales of the waves are much shorter than those of the mean flow. The wave-mean flow interaction has been extensively studied for the cases when the mean flow is prescribed externally—as a stationary or time-dependent current (a "potential barrier").

In this talk, I will describe a new type of the wave-mean flow interaction whereby a short-scale wave projectile—a soliton or a linear wave packet—is incident on the evolving large-scale nonlinear dispersive hydrodynamic state: a rarefaction wave or a dispersive shock wave (DSW). Modulation equations are derived for the coupling between the soliton (wavepacket) and the mean flow in the nonlinear dispersive hydrodynamic state. These equations admit particular classes of solutions that describe the transmission or trapping of the wave projectile by an unsteady hydrodynamic state. Two adiabatic invariants of motion are identified in both cases that determine the transmission, trapping conditions and show that solitons (wavepackets) incident upon smooth expansion waves or compressive, rapidly oscillating DSWs exhibit so-called hydrodynamic reciprocity. The latter is confirmed in a laboratory fluids experiment for soliton-hydrodynamic state interactions.

The developed theory is general and can be applied to integrable and non-integrable nonlinear dispersive wave equations in various physical contexts including nonlinear optics and cold atom physics. As concrete examples we consider the Korteweg-de Vries, defocusing Nonlinear Schrödinger and the viscous fluid conduit equations. The talk is based on recent papers [1-3].

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NONLINEAR GENERATION OF FLOWS OF ULTRARELATIVISTIC CHARGED PARTICLES BY ELECTROMAGNETIC WAVES IN THE SPACE PLASMA

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The ultrarelativistic surfatron acceleration of charged particles including electrons, protons in a cosmic plasma (plasma of interstellar clouds) is studied in the interaction of charged particles with electromagnetic waves that are distributed perpendicular to an external magnetic field. The analysis was carried out for the interaction of particles with two waves that have fairly close frequencies. At the same time, their phase velocities have a small difference and resonant interaction with a high energy exchange in a wave-particle system is possible. The problem is solved on the basis of numerical calculations of a non-linear second-order differential equation for the phase of one of the waves on the trajectory of a charged particle. The influence of the second wave on the strong relativistic acceleration of particles trapped by the first wave, depending on the magnitude of the detuning of the frequencies of the waves, i.e. their phase velocities, as well as the amplitudes of the electric field and the values of the initial phases of the waves. This problem is of interest for the interpretation of experimental data on variations in the flux of cosmic rays, as well as for the correct interpretation of the observed CR variations taking into account the variability of cosmic weather. Based on numerical calculations, the optimal conditions for the capture of particles by the first wave and their long retention in the ultrarelativistic acceleration mode by the first wave were studied for various values of the initial parameters of the problem, as well as the effect of the second wave on the long-term retention of charges by the first wave. Calculations show that with a small but significant difference in the phase velocities of both waves, the second mode does not prevent strong acceleration of charged particles trapped by the first mode as long as its amplitude is above the threshold (for the implementation of surfing) values. In calculations, the surfatron acceleration of charged particles by the first mode can lead to an increase in their energy by four or more orders of magnitude, depending on the parameters of the waves and particles. The complex dynamics of the components of the velocity and momentum of captured particles, as well as the characteristics of both wave modes, are investigated. From a practical point of view, the study is necessary to determine the most favorable parameters of the considered system, at which it will observe ultrarelativistic acceleration of particles up to energies of the order of TeV and even much higher. It is also important to consider in the future the influence of plasma inhomogeneities and magnetic fields.

NO WEAK TURBULENCE FOR OLD MEN

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Here we compute the partition function and the entropy of the weakly interacting wave field in thermal equilibrium and in turbulent energy cascade. Non-surprisingly, the entropy is stationary at equilibrium, while non-equilibrium steady state constantly generates entropy and thus requires its removal by the environment. We then analyze the mutual information between large and small scales, which only appears due to non-linear interaction. This mutual information is found to be finite in thermal equilibrium and grow linearly in time in turbulence. That growth is related to creation of singular non-equilibrium measures - turbulent analogs of Sinai-Ruelle-Bowen measures in dynamical systems. The surprising conclusion is that no-matter how small is the nonlinearity and how close is then the overall statistics to Gaussian, the correlation between the upper and lower parts of the cascade, as measured by the mutual information, grows non-stop. That has profound implications for our fundamental understanding of turbulence

and for the practical issue of representation of subgrid scales in turbulence modeling — we suggest that the quality of such modeling must be measured, among other things, by the mutual information, that is information capacity of turbulent cascade.

INTEGRABLE LAGRANGIANS AND PICARD MODULAR FORMS

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We consider first-order Lagrangians $\int f(v_{x_1}, v_{x_2}, v_{x_3})\, dx_1 dx_2 dx_3$ such that the corresponding Euler-Lagrange equations

$$(f_{v_{x_1}})_{x_1} + (f_{v_{x_2}})_{x_2} + (f_{v_{x_3}})_{x_3} = 0$$

belong to the class of 3D dispersionless integrable systems. It is demonstrated that the generic integrable Lagrangian density f is a Picard modular form of its arguments. Explicit parametrisation of f by generalised hypergeometric functions of Appell's type is obtained. Alternative representations and degenerations of f are also discussed. The talk is based on a joint project with F. Cléry, A Odesskii and D. Zagier.

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FORCED BOUSSINESQ-TYPE MODELS FOR NONLINEAR STRAIN WAVES IN SOLID WAVEGUIDES

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The study of nonlinear waves in solids is an important theme of the current research on waves. The research includes the propagation of bulk strain solitons in solid waveguides. Historically, theoretical developments began with the studies of waves in elastic rods of circular cross section [1-4].

In our work we systematically derive long wave models for weakly-nonlinear longitudinal bulk strain waves in rods of circular cross section within the scope of the Murnaghan model of the weakly nonlinear dynamic elasticity. In contrast to the previous studies we derive Boussinesq-type equations without any simplifying assumptions on the Lagrangian of the problem, and add axisymmetric loading on the boundary surface and longitudinal prestretch with the view of modelling localised waves observed in experiments [5]. We derive two forced Boussinesq-type models from the full equations of motion and non-zero surface boundary conditions, utilising the presence of two small parameters characterising the smallness of the wave amplitude and the long wavelength compared to the radius of the waveguide. We also derive the appropriate uni-directional models.

Moreover, we perform numerical simulations of the full problem formulation for a number of model examples, and compare the predictions of our reduced mathematical models with the results of direct numerical simulations. There is good agreement for weakly nonlinear waves.

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A SERIES OF AUTONOMOUS QUAD EQUATIONS

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We present an infinite series of autonomous discrete equations on the square lattice

$$(u_{n,m+1}+1)(u_{n,m}-1)=\beta_N(u_{n+1,m+1}-1)(u_{n+1,m}+1),$$

where β_N is primitive root of unit, i.e.

$$\beta_N^N=1, \qquad \beta_N^j\neq 1 \text{ for all } 1\le j<N.$$

We show that equations of this series possess hierarchies of autonomous generalized symmetries and conservation laws in both directions. Their orders in both directions are equal to κN, where κ is an arbitrary natural number and N is equation number in the series. Such a structure of hierarchies is new for discrete equations in the case $N>2$.

Symmetries and conservation laws are constructed by means of the master symmetries. Those master symmetries are found in a direct way together with generalized symmetries. Such construction scheme seems to be new in the case of conservation laws. One more new point is that, in one of directions, we introduce the master symmetry time into coefficients of discrete equations.

In most interesting case $N = 2$ the simplest autonomous generalized symmetries read:

$$\partial_{t_2} u_{n,m} = (u_{n,m}^2 - 1)\big[(u_{n,m+1}^2 - 1)(u_{n,m+2} + u_{n,m}) - (u_{n,m-1}^2 - 1)\times \\ \times(u_{n,m} + u_{n,m-2}) - 4(u_{n,m+1} - u_{n,m-1})\big],$$

$$\begin{aligned} \partial_{\theta_2} u_{n,m} =& (u_{n,m}^2 - 1)(T_n - 1)\left(\frac{(u_{n+1,m} + u_{n,m})}{U_{n,m}} + \frac{(u_{n-1,m} + u_{n-2,m})}{U_{n-1,m}}\right), \\ U_{n,m} =& (u_{n+1,m} + u_{n,m})(u_{n,m} + u_{n-1,m}) - 2(u_{n,m}^2 - 1). \end{aligned}$$

We show that in this case a second order generalized symmetry in n-direction is closely related to a relativistic Toda type integrable equation. As far as we know, this property is very rare in the case of autonomous discrete equations.

For more details see [1].

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STRONGLY INTERACTING SOLITON GAS

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Statistical behavior of nonlinear integrable systems, called in general integrable turbulence [1], is a rapidly developing area of theoretical and experimental studies. Here we consider integrable turbulence of soliton gas of high spatial density, i.e. strongly interacting soliton gas. To model it we use N-soliton solutions (N-SS) of the integrable focusing one-dimensional nonlinear Schrodinger equation. This approach first suggested in [2] allows us to create highly nonlinear wave fields, which consist of solitons placed very close to each other to interact strongly. To reveal statistical properties of the soliton gas we generate large statistical ensembles ($\sim 10^3$ realizations each) of the N-SS with $N \sim 10^2$. We study the major statistical characteristics of the wave field, such as the kinetic and potential energies, the kurtosis, the wave-action spectrum and the probability density function of wave field intensity. First, we consider soliton gas having broad distribution of soliton velocities [1]. We show how the properties of such soliton gas change upon increasing soliton density for several types of soliton amplitude and velocity distributions. Then we describe our new results for the case when solitons have narrow velocity distribution or have no velocities at all, that corresponds to the semi-classical approximation in the theory of inverse scattering transform. We focus on the case of fundamental importance – the statistically stationary state of spontaneous noise-driven modulation instability of a condensate background.

We demonstrate how the N-SS can be used to describe statistical characteristics of the modulation instability obtained recently numerically and experimentally in [3] and [4].

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ON STABILITY OF KOLMOGOROV SPECTRA FOR SURFACE GRAVITY WATER WAVES

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The surface water waves interact nonlinearly via the four-wave interactions [1]. These interactions result in the cascading of the energy and the wavenumber via the Kolmogorov spectra [2]. The quadruplet interactions may be integrated to obtain the Kolmogorov spectra parameters [3].

We introduce the small perturbations of special form (based on the logarithm of the frequency) into the Kolmogorov spectra. The integration by quadruplets allows us to calculate the decrement for the damping of these pertubations and show that no perturbation grows with time. This confirms the stability of the Kolmogorov spectra.

The value of the decrement of the perturbation is of importance for the numeric methods of the wave spectrum calculations. This value limits the calculation time step: too large step leads to numeric instability. We caclculate the decrement for different numeric schemes. We show that the DIA method has some innaturally small values for the decrement.

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ON DRESSING FACTORS AND SOLITON SOLUTIONS OF 2-DIMENSIONAL TODA FIELD THEORIES

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In 1981 A. V. Mikhailov [1] introduced the reduction group for soliton equations and as a result discovered the class of 2-dimensional Toda field theories (2d-TFT) related to the algebras $sl(n)$. Soon after that it was established that 2d-TFT can be related to each simple Lie algebra [2] and also to Kac-Moody algebras [3].

The Lax representations of 2d-TFT related to the simple Lie algebra $\mathfrak{g}$ graded by its Coxeter automorphism C is [2]:

$$\begin{aligned} L\psi &\equiv i\frac{\partial\psi}{\partial x} + (U_0(x,t) - \lambda U_1(x,t))\psi = 0, \qquad U_0 = -\frac{\partial\phi}{\partial t}, \quad U_1 = \sum_{\alpha\in A} e^{\phi_\alpha} E_\alpha \\ M\psi &\equiv i\frac{\partial\psi}{\partial t} + (V_0(x,t) - \lambda^{-1} V_1(x,t))\psi = 0, \quad V_0 = \frac{\partial\phi}{\partial x}, \qquad V_1 = \sum_{\alpha\in A} e^{\phi_\alpha} E_{-\alpha}. \end{aligned} \tag{1}$$

Here the real-valued function $\phi(x,t) \in \mathfrak{h}$ and A is the set of admissible roots of $\mathfrak{g}$, see [2]. The Lax pair (1) possesses $\mathbb{D}_h$ as Mikhailov reduction group where h is the Coxeter number of $\mathfrak{g}$, $C^h = 1$. The corresponding 2d-TFT take the form:

$$2\frac{\partial\phi}{\partial x\partial t} = \sum_{\alpha\in A} \alpha\, e^{2\phi_\alpha(x,t)}. \tag{2}$$

Our aim is to extend the Zakharov-Shabat dressing method [4,5] and to construct dressing factors $u(x,t,\lambda)$ for $\mathbb{D}_h$-graded Lax pairs like (1). This allows us to derive the soliton solutions of the 2d-TFT (2).

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NORMAL FORM OF EQUATIONS WITH NONLINEARITIES OF DISLOCATIONS AND FERMI–PASTA–ULAMA

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The class of Ferm–Pasta–Ulam equations and equations describing the dislocations to which a large number of works are devoted are studied (for example, [1], [2]). These equations are of undoubted interest both in the developmental sense and in theoretical studies, being a prominent representative of integrable equations.

In [2] a model was considered that combines these two equations and a number of issues related to the Painlevé integrability of its solutions were studied. In this paper, for such a model with nonlinearities of dislocations and FPU, local dynamic properties of solutions are investigated. An important feature of the model is the fact that the entire infinite set of characteristic numbers of the equation linearized at zero consists of purely imaginary values. Thus, the critical case of infinite dimension is realized in the problem on the stability of the zero solution. For his research, a special asymptotic construction method is used with varying degrees of accuracy of the so-called normalized equations. Using such equations, we determine the main part of the solutions of the original equation, after which we can construct asymptotics using perturbation theory.

All solutions are naturally divided into two classes: regular solutions that smoothly depend on a small parameter entering the equation and irregular, which are a superposition of functions that oscillate quickly in a spatial variable. For each class of solutions, the areas of such a change in the parameters of the equation for which the principal parts are described by different normalized equations are distinguished. Sufficiently broad classes of such equations are presented, which include, for example, the families of the Schrödinger, Korteweg-de Vries, and other equations. We consider the problem of determining such a set of parameters of the original equation for which the nonlinearity of dislocations and the nonlinearity of the FPU are comparable in strength None of them can be neglected in the first approximation.

It is interesting to note that for regular and irregular solutions such parameter regions are different, and in the second case the corresponding region is much wider.

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DIFFUSION CHAOS AND ITS INVARIANT CHARACTERISTICS

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Some parabolic systems of the reaction–diffusion type exhibit the phenomenon of diffusion chaos [1,2]. Specifically, when the diffusivities decrease proportionally, while the other parameters of a system remain fixed, the system exhibits a chaotic attractor whose dimension increases indefinitely.

Reaction–diffusion systems are parabolic boundary value problems of the form

$$\frac{\partial u}{\partial t} = \nu D \Delta u + F(u), \quad \left.\frac{\partial u}{\partial \vec{n}}\right|_{\partial\Omega} = 0, \tag{1}$$

where Δ is the Laplacian; $u \in \mathbb{R}^k$, $k \geq 2$; $D = \operatorname{diag}\{d_1, \ldots, d_k\}$, $d_j > 0$, $j = 1, \ldots, k$; $\nu > 0$ is a parameter responsible for a proportional decrease in the diffusivities; $\vec{n}$ is the outward normal to the sufficiently smooth boundary $\partial\Omega$ of a bounded domain $\Omega \subset \mathbb{R}^m$, $m \geq 1$; and $F(u)$ is a smooth vector function. It is well known that these systems serve as mathematical models of many biophysical and ecological processes (see [3]). A typical situation is that the point model corresponding to system (1), i.e., the system of ordinary differential equations $\dot{u} = F(u)$, has an exponentially orbitally stable cycle $u = u_0(t)$, $du_0/dt \neq 0$ of period $T_0 > 0$.

It is easy to see that cycle $u_0(t)$ is also exhibited by distributed model (1). In order to examine its stability properties, we obtain the system

$$\dot{h} = [A_0(t) - zD]h, \tag{2}$$

where $A_0(t) = F'(u)|_{u=u_0(t)}$; the parameter z takes the discrete values $\nu\lambda_j$, $j = 0, 1, \ldots$; and $0 = \lambda_0 < \lambda_1 \leq \lambda_2 \leq \ldots$ are the eigenvalues of the operator $-\Delta$ with Neumann boundary conditions arranged in increasing order. Then we assume that z in (2) varies continuously on the half-line $z \geq 0$. Let $\mu_s = \mu_s(z)$, $s = 1, \ldots, k$ denote the multipliers of system (2), and define

$$\alpha(z) = \max_{1 \le s \le k} \left\{ \frac{1}{T_0} \mathrm{Re} \ln \mu_s(z) \right\}. \tag{3}$$

It is always true that $\alpha(0) = 0$.

Below, we need the following definition (see [1,2]).

Definition 1 *The parabolic boundary value problem (1) is said to be biological (complicated) or belong to the class* $\mathbf{B}$ *if the following constraints hold:*

(a) *The point model* $\dot{u} = F(u)$ *has an exponentially orbitally stable cycle* $u_0(t)$.

(b) *There are* $0 \le z_1 < z_2$ *such that function (3) is strictly positive on the interval* $z_1 < z < z_2$.

(c) *For all sufficiently small* $\nu > 0$, *the dynamical system generated by problem (1) in the phase space* $C(\overline{\Omega}; \mathbb{R}^k)$, $\overline{\Omega} = \Omega \cup \partial\Omega$, *has a chaotic attractor* A_ν *whose Lyapunov dimension* $d_L(A_\nu)$ *tends to* $+\infty$ *as* $\nu \to 0$.

Condition (a) is typical of boundary value problems (1) arising in various biophysical and ecological applications. The condition $\alpha(z) > 0$ for $z \in (z_1, z_2)$ in (b) ensures that cycle $u_0(t)$ of distributed model (1) is unstable for all sufficiently small ν. Condition (c) is the most important of the three and guarantees that the phenomenon of diffusion chaos occurs as $\nu \to 0$. Note that, the Lyapunov dimension in (c) can be replaced by the Hausdorff or any other one. For the convenience of the subsequent numerical analysis, we use the Lyapunov dimension, assuming that $d_L(A_\nu)$ is defined in terms of the characteristic numbers of the attractor A_ν by the Kaplan–Yorke well-known formula. Based on Definition 1, the concept of diffusion chaos is easily formulated as follows: *the class* $\mathbf{B}$ *of parabolic systems (1) is not empty.*

So by the term “diffusion chaos” we will mean a strange attractor of boundary problem (1) that nontrivially depends on the spatial variable. At the present time, there are two concepts of diffusion chaos: low-mode and multimode chaos. The first type of chaos can occur in system (1) for average values of the parameter ν, while the second type of chaos occurs for $\nu \to 0$. A numerical analysis of multimode diffusion chaos is carried out in [1,2,4] with the following important conclusion: if the diffusion chaos occurs in system (1) for $\nu \to 0$, then the Lyapunov dimension of the chaos increases without limit.

In this article various finite-dimensional models of diffusion chaos were considered that represent chains of coupled ordinary differential equations and similar chains of discrete mappings. A numerical analysis suggests that these chains exhibit chaotic attractors of arbitrarily high dimensions.

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ON THE DERIVATIVE NONLINEAR SCHRÖDINGER EQUATION RELATED TO SYMMETRIC SPACES

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We will present multi-component generalizations of derivative nonlinear Schrödinger (DNLS) type of equations having quadratic bundle Lax pairs related to $\mathbb{Z}_2$-graded Lie algebras and **A.III** symmetric spaces. The Jost solutions and the minimal set of scattering data for the case of local and nonlocal reductions are constructed.

Furthermore, the fundamental analytic solutions (FAS) are constructed and the spectral properties of the associated Lax operators are briefly discussed. The Riemann-Hilbert problem (RHP) for the multi-component generalizations of DNLS equation of Kaup-Newell (KN) and Gerdjikov-Ivanov (GI) types is derived. A modification of the dressing method is presented allowing the explicit derivation of the soliton

solutions for the multi-component GI equation with both local and nonlocal reductions.

The infinite set of integrals of motion for these models is briefly described at the end.

Based on a joint work with Vladimir Gerdjikov and Rossen Ivanov.

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PERIODIC NLS CAUCHY PROBLEM FOR THE ROGUE WAVES

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We construct an approximate solution for the periodic Cauchy problem for the focusing Nonlinear Schrodinger equation in terms of elementary functions assuming that that the Cauchy data is a small perturbation of the unstable background. This problem is used as a basic nonlinear model of rogue waves generation due to the modulation instability. The formulas obtained by the authors were recently used in optical experiments dedicated to the Fermi-Pasta-Ulam-Tsingou recurrence of anomalous waves.

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CLASSIFICATION OF INTEGRABLE TWO-DIMENSIONAL LATTICES VIA LIE-RINEHART ALGEBRAS

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In the talk a classification method for nonlinear integrable equations with three independent variables will be discussed based on the notion of the integrable reductions. We call the equation integrable if it admits a large class of reductions being Darboux integrable systems of hyperbolic type equations with two independent variables, [1]. The hyperbolic type system is Darboux integrable if and only if its both characteristic Lie-Rinehart algebras are of finite dimension [2]. The most natural and convenient object to be studied in the frame of this scheme is the class of two dimensional lattices generalizing the well-known Toda lattice. We deal with the class of quasilinear equations of the form

$$u_{n,xy} = \alpha_n u_{n,x} u_{n,y} + \beta_n u_{n,x} + \gamma_n u_{n,y} + \delta_n,$$

where α_n, β_n, γ_n, δ_n are functions of u_{n+1}, u_n, u_{n-1}. In [3] some classification results are obtained, new examples are presented.

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THE FOKAS METHOD AND INTEGRABLE NONLINEAR PDEs IN TIME-DEPENDENT DOMAINS

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The *Fokas Method*, or *Unified Transform Method*, is the appropriate generalization of the classical inverse scattering method which renders it applicable to the context of *initial-boundary value problems (IBVPs)* (as opposed to *initial value problems*) for integrable nonlinear evolution PDEs in 1+1 (i.e., one spatial and one temporal) dimensions. This method has been applied, with great success, to the analysis of such problems on the *half-line*, as well as on a *finite interval* of the spatial variable. In this work, we extend the Fokas method by presenting a general formalism for analyzing *moving IBVPs*, i.e., *IBVPs* in *time-dependent domains*, for integrable nonlinear evolution PDEs in $1+1$ dimensions. We consider both cases of *one-point*, as well as *two-point*, *time-dependent domains* and show that the analysis can be reduced to a *moving Riemann-Hilbert problem* in the complex domain of the so-called spectral parameter. As an application, we demonstrate that this methodology can be applied to the cases of *modified Korteweg-de Vries (mKdV)*, *Korteweg-de Vries (KdV)*, *sine-Gordon*, and *modified Nonlinear Schrödinger (mNLS)*, equations.

BOUNCING OF BINARY MATTER WAVES ON REFLECTING BARRIERS IN THE PRESENCE OF GRAVITY

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In the framework of quasi-1D Gross-Pitaevskii equation we consider two matter-wave solitons, performing bouncing dynamics above the reflecting barrier (atomic mirror) under the action of gravity. A variational approach has been developed which allows to find the stationary state of the system, and small amplitude dynamics near the stationary state.

Bouncing dynamics of particles and wave packets above the reflecting surface under the action of gravity has been widely studied, and often employed to illustrate variety of fundamental physical effects [1,2]. For instance, very slow ultracold neutrons totally reflect on the perfectly polished horizontal surfaces and perform periodic motion due to returning force of gravity. This phenomenon has allowed to observe for the first time the quantum states of ultracold neutrons in the gravitational potential of the Earth [3]. Extensions in the context of matter-wave solitons were reported in a number of publications [4 - 8]. Our objective in this work is to consider, using analytic and numerical methods, the bouncing dynamics of two anti-phase bright solitons above the reflecting barrier in presence of gravity. At first we develop a variational approach to find the stationary state of the system, namely the waveforms of solitons and the distance from the barrier at which solitons stay motionless (levitate). After the stationary state is found, we excite the dynamics by slightly perturbing the system through the strength of gravity, or periodically changing the coefficient of nonlinearity. By slowly changing the frequency of modulations of the barrier's position, or the coefficient of nonlinearity, we reveal resonant response of the soliton at a set of particular frequencies.

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INTERACTIONS OF COHERENT STRUCTURES ON THE SURFACE OF DEEP WATER

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In the work [1], the dynamics of pairwise interactions of coherent structures (breathers) on the surface of deep water were numerically investigated in the framework of the Dyachenko-Zakharov model. Significant differences were found in the collision dynamics of breathers of the compact Dyachenko-Zakharov equation when compared to the behavior of the NLSE solitons. It was revealed that an important parameter determining the dynamics of pairwise collisions of breathers is the relative phase of these objects at the moment of interaction.

We perform numerical simulation of the breather interactions by using fully nonlinear 2D Euler equations written in conformal variables. To generate initial conditions in the form of separate breathers we use the reduced model– Zakharov equation. We use explicit expression for the the four-wave interaction coefficient and third order accuracy formulas to recover physical variables in the Zakharov model. The suggested procedure allows to generate breathers of controlled phase which propagate stably in the fully nonlinear model demonstrating only minor radiation of incoherent waves. We perform detailed study of breather collision dynamics depending on their relative phase. We show that all effects found in the work [1] can be observed in the fully nonlinear model. Namely, we report that the relative phase controls the process of energy exchange between breathers; level of energy loses and magnitude and sign of the space shifts that breathers acquire after the collision. The results of our numerical experiments in both Dyachenko-Zakharov and exact models demonstrate similar dynamics of breather interactions, which indicates that the theoretical picture of the interaction of coherent structures presented here is universal and can be observed in laboratory experiments.

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ASYMPTOTICS OF THE DYNAMIC BIFURCATION SADDLE-NODE

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It Is considered a differential equation of the second order, parameters which are slowly vary. Under frozen parameters the corresponding autonomous equation has an equilibriums: saddle and stable nodes. When parameters are deforming pair saddle-node is joining. Asymptotic solution is constructed near such dynamic bifurcation. It Is discovered that in a narrow transient layer the main term of asymptotics is determined by Rikkati and KPP equations. The main result is a determination of the shift of the transient layers from moment of the bifurcations. Exact statement are illustrated by numerical experiments.

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A FIBRE-OPTIC COMMUNICATION SYSTEM USING INVERSE SCATTERING TRANSFORM BASED ON SOLVING A RIEMANN-HILBERT PROBLEM

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Fibre-optic communication is responsible for the major fraction of the global data transfer network. The everyday increase of demand for more data rate has made it essential to come up with new ideas and strategies which are specially tailored to the physical characteristics of the fibre. The fibre-optic communication channel is different from the radio and other traditional channels by its nonlinear features. Within certain limits the light evolution in fibre can be modelled by the nonlinear Schrödinger equation (NLSE) which in its dimensionless form is

$$iq_z + q_{tt} + 2|q|^2 q = 0, \tag{1}$$

where $q = q(t, z)$ is the slow-varying envelope of light, z is the distance along the fibre, and t is the retarded time. In a communication system an input signal, modulated with the random data to be sent to the receiver, undergoes linear and nonlinear distortions in optical fibre. These perturbations of the signal make it difficult at the receiver to realise what the original transmitted random data was. This difficulty mainly attributes to the nonlinear interaction of different signal components while the linear evolution can be easily resolved. Thanks to the special characteristics of the NLSE, it is possible to find some features of the signal (as the boundary condition of the NLSE) with linear and possibly trivial dynamics as the signal travels in the fibre. This special aspect is that the NLSE (1) is integrable via inverse scattering transform (IST) technique. In the IST paradigm an auxiliary ordinary linear differential equation, the Zakharov-Shabat system (ZSS) for the NLSE, plays a central role. The spectrum of this auxiliary equation makes up a set which parametrises the NLSE solutions. This set is called the *nonlinear spectrum* (NS). Nonlinear spectrum can be defined for two types of NLSE solutions; decaying and finite-gap solutions. In the former, the solution has vanishing boundaries, and the latter is attributed to a ZSS whose spectrum has only a finite number of gaps (or arcs).

What makes the NS such a great candidate to modulate in order to convey data is its linear evolution in fibre. Not only it helps using a wide range of already-developed techniques for linear communication, but it also provides better control over some important traits of the signal in fibre. This idea was proposed many years ago [1] where a simple solitonic signal carries data on a part of its NS. This system is only a limited version of what can be achieved when the whole NS is used. The potentials of such a system have been shown in simulations and also through experimental demonstrations [2, and the references therein]. However, most of the research in this area is with the vanishing boundary condition assumption while it has been shown that periodic signals can offer more favourable properties [3]. This advocates employing finite-gap theory to construct periodic signals. A periodic finite-gap solution is parametrised with an NS comprising of a static part (*main spectrum*) and a dynamic part (*auxiliary spectrum*). As the signal propagates in the fibre (along z), the auxiliary spectrum travels over a Riemann surface which is made according to the main spectrum. A complicated system of differential equations governs the evolution of the auxiliary spectrum on this Riemann surface but an Abel map, defined on this surface, can simplify it. Having this simplified dynamic, it is possible to find the z-dependent NS and construct the 2-dimensional solution to (1). In order to make up a finite-gap solution to the NLSE the associate ZSS:

$$\mathbf{\Phi}_t = \begin{bmatrix} -i\lambda & q(t,z) \\ -q(t,z)^* & i\lambda \end{bmatrix} \mathbf{\Phi}, \tag{2}$$

needs to be solved and a Bloch solution, $\mathbf{\Phi}$, needs to be constructed. Two linearly independent matrix solutions of (2) with their corresponding asymptotic behaviour:

$$\mathbf{\Phi}^{\pm}(t;\lambda) = \begin{pmatrix} e^{-i\lambda t} & 0 \\ 0 & \pm e^{-i\lambda t} \end{pmatrix} \mathbf{as}\, t \to \pm\infty, \tag{3}$$

span the solution space and a transition matrix, $M(\lambda)$, independent of t exists where $\mathbf{\Phi}^{-}(t;\lambda) = \mathbf{\Phi}^{+}(t;\lambda)M(\lambda)$. It is possible to rearrange the elements of $\mathbf{\Phi}^{\pm}(t;\lambda)$ to form a Riemann-Hilbert problem (RHP) over an oriented contour $\Gamma \in \mathbb{C}$ with a jump matrix $J(\lambda)$; $\mathbf{\Psi}^{-}(\lambda) = \mathbf{\Psi}^{+}(\lambda)J(\lambda)$ for $\lambda \in \Gamma$, $\mathbf{\Psi}^{-}(\infty) = I$. Solving this problem yields a solution to (1) as

$$q(t,z) = 2i \lim_{|\lambda|\to\infty} \lambda \mathbf{\Psi}(\lambda)_{12}. \tag{4}$$

By establishing the inverse transformation stage (i.e. making up a signal starting from an NS) as an RHP, we can numerically construct the solution using (4) without resorting to the computationally expensive Riemann theta function. This is the advantage of solving an RHP compared to the standard algebro-geometric approach [4].

The main and auxiliary spectrum, which form the jump matrix, $J(\lambda)$, can carry information (mapped on them at the transmitter) to the receiver in a fibre-optic communication system. At the receiver, the NS is calculated, and the transmitted data is retrieved. We have demonstrated through numerical simulations of a signal with only one cut in its spectrum, that a system based on out approach delivers quality performance. The performance of a communication system is measured through its rate of data transmission and error. We evaluated the system performance and investigated its behaviour for different signal powers and fibre distances and showed its competence. In particular, we can achieve a mutual information of 7.6 bits/symbol up to 720 km of noisy fibre link with lumped amplification, i.e. with using a path-averaged model considering an NLSE with periodic loss and gain. This is attained by only utilising the main spectrum and ignoring the auxiliary one which can further increase the system throughput. However, there are still some challenges such as numerical complexity and signal characteristics which need to be improved. One of these characteristics is the number of data symbols carried by each signal which is equivalent to the number of jumps (or arcs) in the RHP. Increasing this number makes it difficult to make sure the signal is periodic in the time domain. Hence, the most challenging next step is to design an inverse transformation scheme by solving an RHP which can incorporate several arcs and control the periodicity and the period of the signal.

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DISPERSIVE SHOCK WAVE THEORY FOR NONINTEGRABLE EQUATIONS

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Although the averaging Whitham method [1] can be applied to description of slow modulations of periodic solutions of any nonlinear wave equation, it is especially effective in the case of completely integrable equations only and the general enough statements are rather scarce. The most important general statement in the form of the 'number of waves conservation law' was given by Whitham himself in Ref. [1]. Application of Whitham method to dispersive shock waves (DSWs) theory initiated in Ref. [2] led Gurevich and Meshcherkin [3] to the conjecture that the value of one of the Riemann invariants of a dispersionless approximation is transferred through the DSW under consideration. Combining this conjecture with Whitham's number of waves conservation law, El developed the method [4] which allows one to calculate velocities of the edges of DSWs for a very wide class of nonintegrable wave equations in a particular case of step-like initial conditions. In this talk, we show that generalization of a remark made in Ref. [5] for the KdV equation case permits us [6] to extend El's method to description of evolution of DSWs formed after wave breaking of simple waves propagating into quiescent medium what greatly widens the practical applicability of the Whitham theory to description of DSWs. This new method is illustrated by applications to several typical nonintegrable wave equations. The analytical theory is confirmed by numerical simulations.

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REDUCING A FREE-BOUNDARY PROBLEM TO THE SYSTEM OF DIFFERENTIAL EQUATIONS

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It is known that the Kirchhoff method allows one to find exact solutions for steady plane potential flows of an ideal fluid with a free boundary in the absence of the gravity force and surface tension. There are many jet flows of this type known today. However, if the problem is more complicated, for example,

- if the flow is ponderous, i.e., the gravity force is taken into account,
- or if the problem is unsteady, i.e., the time is taken into account,

then there is no universal method for constructing exact solution in both cases.

In the present work, we propose the technique to obtain exact solutions basing on analytical continuation of the unknown function beyond the area of its definition. At first, by using conformal mapping the problem is formulated in the form of boundary-value problem in the fixed domain. As domains, we used a wedge with an apex angle α or the strip of unit width. After multiple turns around the wedge apex or after analytical continuation across the strip, various branches of the unknown function are related by an infinite system of differential equations.

It has been demonstrated that the system of equations becomes finite:

- if α/π is a rational number,
- if the function is periodic across the strip with a rational period.

By using the proposed technique, some of exact solutions have been obtained for stationary flows of heavy liquid, which turned out to be very close to high-amplitude gravity waves on the fluid surface. An exact solution simulating the water-fall flow has been constructed. A new class of self-similar flows with a free boundary has been found.

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RELAXATION CYCLES IN A MODEL OF TWO WEAKLY COUPLED GENERATORS WITH DELAYED SIGN-CHANGING FEEDBACK

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Consider a system of two differential equations with delay

$$\begin{aligned}\dot{u}_0 + u_0 &= \lambda F(u_0(t-T)) + \gamma_1/\ln\lambda(u_1 - u_0),\\ \dot{u}_1 + u_1 &= \lambda F(u_1(t-T)) + \gamma_1/\ln\lambda(u_0 - u_1).\end{aligned} \tag{1}$$

Here, u is a scalar function, delay parameter T, coefficient λ, and parameter γ_1 are positive, $F(u)$ is compactly supported function, i.e. for some $p > 0$ we have equality

$$F(u) = \begin{cases} f(u), & |u| < p,\\ 0, & |u| \geq p.\end{cases} \tag{2}$$

We assume that function $f(u)$ is bounded, piesewise smooth and

$$\begin{cases} f(u) \neq 0 & \text{for almost all} \quad u \in (-p, p), \\ f''(p) \neq 0, & f''(-p) \neq 0. \end{cases} \tag{3}$$

System (1) under conditions (2) and (3) simulates two weakly coupled generators with delayed feedback [1,2]. Such generators are used in the manufacture of D-amplifiers, sonars, noise radars [2].

We study nonlocal dynamics of the model (1) under conditions (2), (3) and under assumption that parameter λ is sufficiently large

$$\lambda \gg 1. \tag{4}$$

Using special method of large parameter [3] we reduce investigating of existence and stability of relaxation periodic solutions of initial infinite dimensional system to investigating of existence and stability of rough cycles of some three-dimensional mapping. This mapping have sign-preserving and sign-changing rough stable cycles.

It is shown that system (1) has stable sign-preserving and sign-changing relaxation periodic solutions with amplitude of the order $O(\lambda)$ and period of the order $O(\ln \lambda)$ as $\lambda \to \infty$. Asymptotic formulas of these solutions were constructed. Note, that characteristics of solutions do not depend on exact view of nonlinear function $f(u)$ (they depend on integral properties of function $f(u)$ only). The phenomenon of multistability in system (1) under conditions (2), (3), and (4) is shown.

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SPATIALLY INHOMOGENEOUS SOLUTIONS OF THE SYSTEM WITH A DEVIATION IN SPACE

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Consider parabolic equation with deviation of spatial variable

$$\frac{\partial u}{\partial t} + u = \varepsilon \frac{\partial^2 u}{\partial x^2} + K \sin u(t, x - h) \tag{1}$$

and periodic boundary conditions

$$u(t, x + 2\pi) \equiv u(t, x). \tag{2}$$

Here $0 < \varepsilon \ll 1$, $K \in \mathbb{R}$. Value h describes the deviation of the spatial variable (rotation of the field at an angle of h). Let h be close to rationally proportional to 2π number, i.e. for some coprime m_1 and m_2

$$h = 2\pi \frac{m_1}{m_2} + \mu,$$

where μ is another small parameter: $0 < \mu \ll 1$.

Let u_0 be uniform equilibrium of (1), (2): $u_0 = K \sin u_0$. The problem is to investigate the behavior of solutions (1), (2) for $t \geq 0$ in some sufficiently small (but fixed) neighborhood of u_0.

Denote $p = K \cos u_0$. If $|p| < 1$ then the behavior of solutions with initial conditions from some neighborhood of u_0 is trivial: all of them tends to u_0. If $|p| > 1$ then almost all solutions from some neighborhood of u_0 leave it. The dynamics is nonlocal. Other two cases are critical.

The most interesting case occurs when the parameter p is close to -1. So for some small ν we have $p = -1 - \nu$.

Thus, the problem contains three small parameters at once: ε, μ and ν. Their ratio is very important and has a significant impact on the results and the course of research.

In the critical case under consideration, the real parts of the infinite set of roots of the characteristic equation tend to zero as $\varepsilon, \mu, \nu \to 0$. Thus, we can say that the realizable critical case has infinite dimension.

The main result of the work is that the original problem in the case under study is reduced to a so-called. the quasinormal form – a family of nonlinear equations independent of small parameters whose solutions give the main parts of the asymptotic approximation of the solutions of the original problem that is uniform over all $t \geq 0$ [1,2].

For example, let $h = \pi + \varepsilon^{1/2} h_1$, $p = -1 - \varepsilon p_1$. In this situation the quasinormal form is

$$\frac{\partial xi}{\partial \tau} = \left(1 + \frac{1}{2} h_1^2\right) \frac{\partial^2 \xi}{\partial r^2} + p_1 \xi - \left(\frac{1}{6} + \frac{u_0^2}{4}\right) \xi^3 \tag{3}$$

with antiperiodic boundary conditions

$$\xi(\tau, r + \pi) \equiv -\xi(\tau, r). \tag{4}$$

Theorem 1. *Let ξ be bounded with its derivatives solution of (3), (4). Then*

$$u(t,x,\varepsilon) = u_0 + \varepsilon^{1/2} \xi(\varepsilon t, x - \varepsilon^{1/2} h_1 (1 + O(\varepsilon^{1/2}))t) - \varepsilon \frac{1}{2} u_0 \xi^2 (\varepsilon t, x - \varepsilon^{1/2} h_1 (1 + O(\varepsilon^{1/2}))t)$$

produce asymptotically small residual of order $O(\varepsilon^{3/2})$ uniformly for all $t \geq 0$.

In the situation $h = \pi + \varepsilon^{1/2} h_1$, $p = -1 - \varepsilon^\gamma p_1$ $(0 < \gamma < 1)$ we have a family of equations like (3)

$$\frac{\partial \xi}{\partial \tau} = \varkappa^2 \left(1 + \frac{1}{2} h_1^2\right) \frac{\partial^2 \xi}{\partial r^2} + p_1 \xi - \left(\frac{1}{6} + \frac{u_0^2}{4}\right) \xi^3 \tag{5}$$

with boundary conditions (4). Here $\varkappa$ is arbitrary real parameter. Its solutions gives main part of rapidly oscillating solution of initial problem.

Theorem 2. *Let ξ be bounded with its derivatives solution of (5), (4). Then*

$$u(t,x,\varepsilon) = u_0 + \varepsilon^{\gamma/2} \xi(\varepsilon^\gamma t, r) - \varepsilon^\gamma \frac{1}{2} u_0 \xi^2 (\varepsilon^\gamma t, r)$$

produce asymptotically small residual of order $O(\varepsilon^{3\gamma/2})$ uniformly for all $t \geq 0$. Here $r = (\varkappa \varepsilon^{-(1-\gamma)/2} + \theta_\varkappa)(x - \varepsilon^{\gamma/2} h_1 \varkappa t)$.

If $h = \pi + \varepsilon^\alpha h_1$ $(0 < \alpha < 1/2)$ then quasinormal form getting more complicated. The spatial variable in it become two-dimensional.

$$\frac{\partial \xi}{\partial \tau} = \Delta^2 h_1^{-2} \frac{\partial^2 \xi}{\partial r^2} + \varkappa^2 h_1^2 \frac{\partial^2 \xi}{\partial s^2} + p_1 \xi - \left(\frac{1}{6} + \frac{u_0^2}{4}\right) \xi^3,$$

$$\xi(\tau, r+2\pi, s) \equiv \xi(\tau, r, s) \equiv -\xi(\tau, r, s+\pi).$$

The analog of theorems 1 and 2 is still true.

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INVARIANTS IN SEPARATED VARIABLES: YANG-BAXTER, ENTWINING AND TRANSFER MAPS

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We present the explicit form of a family of Liouville integrable maps in 3 variables, the so-called *triad family of maps* and we propose a multi-field generalisation of the latter. We show that by imposing separability of variables to the invariants of this family of maps, the H_I, H_{II} and H_{III}^A Yang-Baxter maps in general position of singularities emerge. Two different methods to obtain entwining Yang-Baxter maps are also presented. The outcomes of the first method are entwining maps associated with the H_I, H_{II} and H_{III}^A Yang-Baxter maps, whereas by the second method we obtain non-periodic entwining maps associated with the whole F and $H-$list of quadrirational Yang-Baxter maps. Finally, we show how the transfer maps associated with the F and the H lists of Yang-Baxter maps can be considered as the $(k^{th}-1)$-iteration of some maps of simpler form. We refer to these maps as *extended transfer maps* and in turn they lead to $k-$point alternating recurrences which can be considered as the autonomous versions of some hierarchies of discrete Painlevé equations.

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THE EFFECTS OF THE SHEAR FLOWS ON SURFACE AND INTERNAL RING WAVES

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In this talk I will overview some results concerning the effects of the parallel shear flow on long weakly-nonlinear surface and internal ring waves in a stratified fluid (e.g., oceanic internal waves generated in narrow straits and river-sea interaction zones). We showed that despite the clashing geometries of the waves and the shear flow, there exists a linear modal decomposition (separation of variables) in the set of Euler equations describing the waves in a stratified fluid, more complicated than the known decomposition for the plane waves. We used it to describe the wavefronts of surface and internal waves, and to derive a 2D cylindrical Korteweg – de Vries (cKdV)-type model for the amplitudes of the waves. Here, we apply the general theory to two- and three-layered fluids with different types of the shear flow. The distortion of the wavefronts is described explicitly by constructing the singular solution (envelope of the general solution) of a respective nonlinear first-order differential equation. This is joint work with Xizheng Zhang, Curtis Hooper and Noura Alharthi.

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PARAMETRIC ORIGIN OF AN INTRACAVITY SOLITON IN A SUPERRADIANT LASER WITH A LOW-Q CAVITY

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Abstract: We find a new parametric mechanism of the self-mode-locking which exists due to beating of two superradiant modes with a period two times less than a round-trip period and leads to an intracavity soliton formation, without using any mode-locking technique, in a CW non-stationary superradiant laser with a combined distributed-feedback Fabry-Perot low-Q cavity.

Summary: By means of numerical modeling of the Maxwell-Bloch equations, we find an efficient parametric mechanism of the self-mode-locking in a continuous-wave (CW) superradiant laser with a combined distributed feedback (DFB) Fabry-Perot low-Q cavity. The mechanism is based on a quantum-coherence phenomenon (Rabi oscillations) which takes place if an active-center polarization (optical dipole) lifetime exceeds a photon lifetime in the cavity. Namely, we find that for such lasers the intracavity soliton formation is possible without using any mode-locking technique (passive or active) if there are two non-stationary superradiant lasing modes with the frequency spacing two time greater than that of the quasi-equidistant quasi-stationary lasing modes.

The latter condition can be satisfied due to (i) a presence of an appropriate photonic band-gap originated due to the distributed-feedback Bragg structure of the laser cavity and (ii) the fact that the higher-Q superradiant modes are situated at the edges of this photonic band-gap while the lower-Q quasi-equidistant modes are shifted apart from it. The existence of the superradiant modes and their efficient coupling with the neighbouring quasi-stationary modes originates from the collective dynamics of active centers in a medium with a strong inhomogeneous broadening and requires high values of their spatial and spectral density as well as low quality factor of the laser cavity [1 - 3].

A detailed numerical modeling, qualitative analytical theory, and estimations of the laser parameters required for an experimental demonstration of the phenomenon in semiconductor lasers are presented. The novel regime of a self-mode-locking in a superradiant laser is compared with a passive mode-locking regime in a standard laser containing a saturable absorber. It is shown that, in addition to the ultrashort coherent pulse travelling around the laser cavity and generated by the quasi-stationary modes, both non-stationary superradiant modes produce quasiperiodic trains of pulses with different repetition cycles much longer than the cavity round-trip time. Interference between those pulses can produce extremely short coherent pulses as well as change strongly a value of population inversion and even reverse its sign in some spatial-spectral domains of an active medium. Those domains in a superradiant laser play part of a resonant saturable absorber and support spontaneous mode-locking of other lasing modes making it more stable.

The parametric mechanism is demonstrated in Fig. 2 by a typical example of the triple-periodic evolution of the spectral density of active-center polarization at the laser edge. The energy contents of the self-mode-locked pulses, the superradiant mode pulses, and the accompanying non-stationary quasi-continuous radiation may achieve the same order of magnetude. The predicted parametric effect exists for a wide range of laser parameters and may be achieved, e.g., in the CW-pumped multilayered heterostructures with the well-dots or sub-monolayer quantum dots and a lateral Bragg grating, ensuring a proper selection of longitudinal modes by means of the distributed feedback. The suggested lasing regime is promising for numerous applications in the quantum information processing, storage and recovery of quantum and phase information, dynamic spectroscopy, optoelectronics, etc.

The work was supported by the Program of fundamental research of the Presidium of the Russian Academy of Sciences n. 32 "Nanostructures: Physics, Chemistry, Biology, Basic Technology".

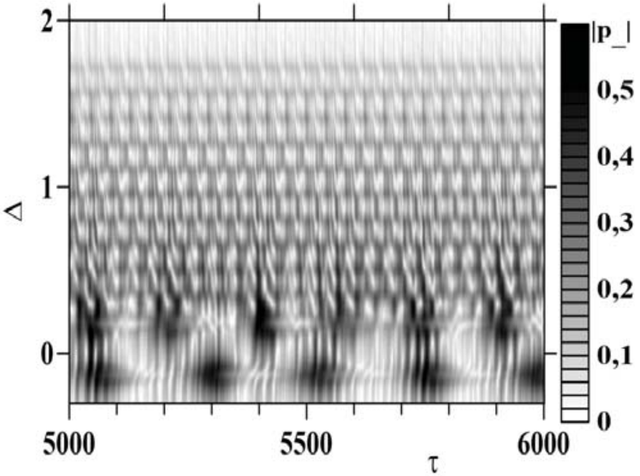

Fig. 2: Typical dynamics of the spectral density of polarization, $|p_{-}(\Delta, \tau)|$, of an active medium at a facet of a low-Q laser cavity with the self-mode-locking owing to the parametric effect of beating between two superradiant modes [3].

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A DENSE LAYER FORMATION IN AN ELECTROSTATIC COLLISIONLESS SHOCK WAVE DURING THE EXPANSION OF A HOT DENSE PLASMA INTO A RAREFIED ONE

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Abstract: The formation and evolution of a dense layer just behind a front of an electrostatic collisionless shock wave in the course of expansion of a dense plasma with hot electrons into a cold background plasma is investigated. The results of the numerical modelling of this phenomenon and the data of the first relevant experiments with a laser plasma expansion are compared. It is demonstrated that the growth and decay of the dense layer is accompanied by the Langmuir, ion-acoustic, and Weibel instabilities as well as a transient appearance of the corresponding soliton-like structures.

Summary: Recent experiments on a thin foils ablation by the femtosecond laser pulses revealed an unexpected dense layer on the slope of a hot plasma expanding into a cold tenuous ionized medium [1-3]. The layer propagates at nearly ion-acoustic velocity and is associated with a complex collisionless behavior of particles in the electrostatic shock wave. We performed the 2D3V and 1D3V numerical particle-in-cell simulations of the Vlasov-Maxwell equations, based on the experimental data and various initial conditions and density profiles. We show that the laser plasma expansion gives rise to a shock wave with a dense layer only if there is a sufficiently large difference in densities as well as inhomogeneous scales of the hot dense laser plasma and the cold rarefied (background) plasma.

The newly observed phenomenon takes place due to the space-time focusing of ion trajectories under the action of the electric field of energetic electrons in the shock front region. By analyzing particle trajectories in the real and phase spaces, we show that the ions from both sides of the initial discontinuity are involved in the formation and maintenance of the dense layer in a traveling shock wave. There are the rarefied plasma ions captured by the shock and the accelerated dense plasma ions that overtake its front, respectively. The contributions of both ion components to the dense layer and its dynamical features under typical laser plasma conditions are discussed (Fig. 3).

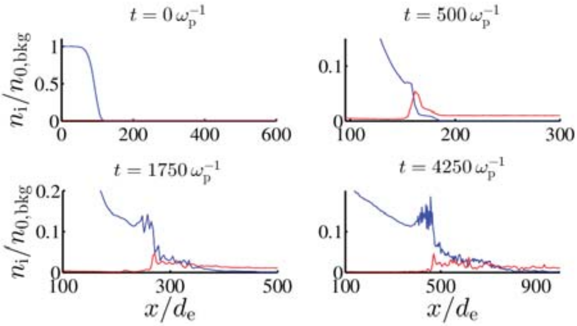

Fig. 3: Spatial distribution of the ion densities in a typical 1D3V particle-in-cell simulations [3] at the four moments of time: blue curves dense plasma ions n_0, red curves background ions nbkg; with m_i/m_e=100. The time and spatial coordinate are given in the units of an inverse plasma frequency and a Debye length of the initial dense laser plasma, respectively. The initial parameters for the dense and background plasmas are max $n_{0i} = 10^{21} cm^{-3}$, $T_e = 2.5$ KeV, $T_i = 3$ eV; and $n_{bkg} = 10^{19} cm^{-3}$, $T_e = 50$ eV, $T_i = 3$ eV, respectively.

At first, the dense layer arises in the background ion density, but soon after it becomes quasistationary the ions from the hot laser plasma start to prevail. They also increase the width of the dense layer behind the shock front. We analyze the conditions for the emergence of a dense layer, the coordinate of its formation, the laws of a slow evolution of its velocity, width and density, its lifetime as well as various phenomena associated with its decay. The latter phenomena include the Langmuir, ion-acoustic, and Weibel instabilities as well as a transient appearance of the corresponding soliton-like structures. In particular, we discuss a formation of the ion-acoustic wave packets which are considerably impacted by a flow of the high-energy electrons from the dense laser plasma. Thus, a scenario of a free solitons formation is challenged.

Soon after the expansion begins, the return electron current supplements the direct one and the magnetic field orthogonal to the simulation plane is generated. Meanwhile the anisotropy of the electron velocity distribution arises behind the shock front triggering the Weibel instability and the growth of an in-plane quasistatic inhomogeneous magnetic field. With the help of PIC-simulations we analyze the evolution of the electron and ion distribution functions and trace the development of the electric and magnetic fields in this strongly inhomogeneous anisotropic plasma. We consider various plasma parameter sets and outline the conditions for a formation of the elongated or flattened electron velocity distributions resulting in the different spatial structures of the magnetic field.

The works was supported by the Russian Foundation for Basic Research (project no. 18-29-21029).

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WAVE TURBULENCE AND COLLAPSES AT THE FREE SURFACE OF A LIQUID DIELECTRIC IN AN EXTERNAL TANGENTIAL ELECTRIC FIELD

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The wave turbulence is observed at interaction of nonlinear dispersive waves in many physical processes. In recent works [1,2], it was found that under certain conditions, regions with a high energy concentration can form at the liquid boundary under the action of strong external tangential electric field. In these regions electrostatic and dynamic pressures undergo a discontinuity. The formation of such discontinuities triggers a direct energy cascade, which leads to the transfer of energy from large scales to small ones. We numerically demonstrate that this process can lead to the development of surface wave turbulence.

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STRUCTURE OF COHERENT VORTICES CAUSED BY THE INVERSE CASCADE OF $2D$ TURBULENCE AND RELATED PROBLEMS

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We consider high-Reynolds two-dimensional forced turbulence that is excited in a finite box. If the bottom friction coefficient is small enough, coherent vortices appear in the ystem as a result of the inverse energy cascade. We establish universal average profile of the cohernt vortices.

UNIVERSAL PARABOLIC REGULARIZATION OF THE GRADIENT CATASTROPHES FOR THE BURGERS-HOPF EQUATION AND JORDAN CHAIN

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Non-standard parabolic regularization of gradient catastrophes for the Burgers- Hopf equation and integrable hydrodynamic type systems with the most degenerate Jordan blocks is considered. An approach is based on the analysis of the generic and all higher order gradient catastrophes and their step by step regularization by embedding the Burgers-Hopf equation and Jordan systems into integrable multicomponent parabolic systems of quasi-linear PDEs with the most degenerate Jordan blocks. Probabilistic realization of such procedure is discussed. The complete regularization is achieved by embedding into infinite Jordan chain. lt is shown that the Burgers equation, Korteweg-de Vries equation and other nonlinear PDEs are particular reductions of the Jordan chain.

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WHY IS THE MICROSTRUCTURE OF THE MAIN PULSE AND INTER PULSE OF THE PULSAR IN CRAB SO STRIKINGLY DIFFERENT?

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In the remarkable work of Hankins and Ailek*) [1] it was discovered (to the surprise of the authors themselves) the striking difference in the spectra of the MP (main pulse) and of the IP (inter pulse) at microsecond resolution. In particular, a wide frequency range was observed in the MP spectra, forming vertical structures, while in the IP spectra at the same frequencies horizontal structures with selected frequencies resembling thezebra structure of solar radio bursts were observed. We offer a possible (partial) answer to this riddle. It consists in, as can be seen from multifrequency measurements (Moffett and Hankins, ApJ, 1996), that in the vicinity of the measurement frequency of the microstructure there is a change in the prevailing mechanisms of radio emission with growing of frequency. The subrelativistic mechanism of radiation with longitudinal acceleration is still valid for the MP (Kontorovich and Flanchik [2]). In this case, a flat in frequency radiation spectrum is formed, which corresponds to the "vertical" microstructures and terminates (breaks off) at the inverse time of particle acceleration. Acceleration occurs to relativistic velocities from thermal ones in the accelerating electric field increasing from zero on the star's surface. For the IP this mechanism has already been turned off at the measurement frequency and replaced by another (relativistic) curvature radiation mechanism, which in an inclined magnetic field causes an IP shift (Kontorovich and Trofimenko [3]). Therefore, there are no vertical microstructures in it. We also considered the effect of a logarithmically large acceleration time from thermal to relativistic velocities (Kontorovich and Flanchik, JETP, 2013) to the frequency spectrum break and the possibility of appearance of the separate MPs with vertical structures due to temperature fluctuations in the region of frequencies larger than the average frequency of emission mechanism change. We do not discuss here the micro zebra structure, the explanation of which are devoted various physical scenarios (see Lyutikov, MN RAS, 2007; Ardavan et al, ibidem, 2008; Zheleznyakov et al, Astron. Lett., 2012; Kontorovich, ibidem, 2014) and whose connection with curvature radiation (through bunches or solitons, as example) is not yet clear.

In traditional pulsar models the MP and IP should be the same in their observable quantities (such as spectrum, time signature, or dispersion). We were and remain quite surprised that this turns out not to be the case in the Crab pulsar. From J. Eilek and T. Hankins paper [1].

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INVERSE CASCADE OF GRAVITY WAVES IN THE PRESENCE OF CONDENSATE: NUMERICAL RESULTS AND ANALYTICAL EXPLANATION

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We consider direct numerical simulation of isotropic turbulence of surface gravity waves in the framework of the primordial dynamical equations. We use approximation of a potential flow of ideal incompressible fluid. System is described in terms of weakly nonlinear equations [1] for surface elevation $\eta(\vec{r}, t)$ and velocity potential at the surface $\psi(\vec{r}, t)$ $(\vec{r} = \overrightarrow{(x, y)})$

$$\begin{aligned}\dot{\eta} =& \hat{k}\psi - (\nabla(\eta\nabla\psi)) - \hat{k}[\eta\hat{k}\psi] + \hat{k}(\eta\hat{k}[\eta\hat{k}\psi]) + \frac{1}{2}\Delta[\eta^2\hat{k}\psi] + \frac{1}{2}\hat{k}[\eta^2\Delta\psi] + \widehat{F}^{-1}[\gamma_k\eta_k],\\ \dot{\psi} =& -g\eta - \frac{1}{2}\left[(\nabla\psi)^2 - (\hat{k}\psi)^2\right] - [\hat{k}\psi]\hat{k}[\eta\hat{k}\psi] - [\eta\hat{k}\psi]\Delta\psi + \widehat{F}^{-1}[\gamma_k\psi_k] + P_{\vec{r}}. \qquad (1)\end{aligned}$$

Here dot means time-derivative, Δ — Laplace operator, $\hat{k}$ is a linear integral operator $\left(\hat{k} = \sqrt{-\Delta}\right)$, $\widehat{F}^{-1}$ is an inverse Fourier transform, γ_k is a dissipation rate, $P_{\vec{r}}$ is the driving term which simulates pumping on small scales. These equations were derived as a results of Hamiltonian expansion in terms of $\hat{k}\eta$ up to the fourth order terms.

Like in works [2, 3] formation of long waves background (condensate) and inverse cascade was observed. This time all invertial interval (range of scales where there is no pumping or damping, only nonlinear interaction of waves) in the inverse cascade region. Currently observed slopes of the inverse cascade are close to $n_k \sim k^{-3.15}$, which differ significantly from theoretically predicted $n_k \sim k^{-23/6} \simeq k^{-3.83}$. In our work we propose some analytical analysis of results, which is in part based on recent works [4, 5].

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DISPERSIONLESS LAX PAIRS: FROM ZAKHAROV AND PENROSE TO NOWADAYS

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Integrability of dispersionless systems in dimensions 3 and 4, with an additional assumption of nondegeneracy, can be interpreted in geometric terms. 25 years ago[1] E. V. Zakharov proposed a formulation of Lax pair using dispersionless limit. For some other systems an apparently diffenrent approach to integrability was developped using the twistor program of R. Penrose. These two approaches are related and are the base for the program “Integrability via Geometry” on which I will report.

I will discuss equivalence of several definitions of dispersionless integrability, as well as classification results for functional classes of dispersionless PDEs. The work is based on multiple collaborations[2,3,4,5].

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LANDAU–HOPF SCENARIO OF PASSAGE TO TURBULENCE CAN BE REALIZED

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In this report, we propose examples where the Landau-Hopf scenario can be realised as a cascade of the Hopf bifurcations [1,2].

Consider the boundary value problem (BVP) [3]

$$w_{tt} + 2\varepsilon(\kappa_1 w_t + \kappa_2 w_{txxxx}) + w_{xxxx} + \varepsilon c^2 w_{xx} + \varepsilon\beta c w_{tx} + \varepsilon\alpha c^2 w' =$$
$$= \varepsilon^2[a_1 \int_0^1 w_x^2 dx + a_2 \frac{1}{2}\frac{d}{dt}\int_0^1 w_x^2 dx] w_{xx}, \tag{1}$$

$$w(t,x) = w(t,1) = w_{xx}(t,0) = w_{xx}(t,1) = 0. \tag{2}$$

Here, ε is small parameter, $\kappa_1, \kappa_2, \beta, \alpha, a_1, a_2 > 0$, c is positive and proportional to the flow velocity. This BVP arises in the elastic stability theory when modeling the dynamics of pipe transporting a liquid with velocity U.

It is possible to reduce the analysis of the dynamics of the solutions for the BVP (1), (3) to the study of a separated system of ordinary differential equations for the amplitude of the Hopf equalities

$$\dot\rho_n = \varepsilon^2[\tau_n(c) - b_n\rho_n^2]\rho_n, \tag{3}$$

where $n = 1, 2, \dots, \tau_1(c) < 0$ for all $c \geq 0$, $\tau_k(c)$ increases with c for $k = 2, 3, 4, \dots$.

For $c \in (c_m, c_{m+1}]$, we have that $\tau_2(c) > 0, \dots, \tau_m(c) > 0, \tau_{m+1}(c) \leq 0, \tau_{m+2}(c) \leq 0$,
$\dots$ ($\tau_1(c) < 0$ always).

Theorem 1. *There exists $\varepsilon_0 > 0$ such that for $\varepsilon \in (0, \varepsilon_0)$ the BVP (1), (2) has torus $T_l(\varepsilon)$ of dimension l, where $l = 1, \dots, m-1$ ($dim T_l(\varepsilon) = l$). If $l < m-1$, then the torus is saddle. The torus $T_{m-1}(\varepsilon)$ is a local attractor.*

It follows from the theorem that only the torus $T_{m-1}(\varepsilon)$, whose dimension is maximal, can be asymptotically stable, if $c \in (c_m, c_{m+1}]$. If c passes from $(c_m, c_{m+1}]$ to $(c_{m+1}, c_{m+2}]$, then the following hold

1) all earlier existing tori are preserved;

2) from each torus $T_l(\varepsilon)$ $(l \leq m-1)$ a new torus $T_{l+1}(\varepsilon)$ of dimension $l+1$ bifurcates;

3) if $l < m-1$, then unstable torus bifurcate from the torus T_l;

4) the torus T_m, whose dimension is maximal for $c \in (c_m, c_{m+1}]$, is the only stable torus.

For solutions belonging to the torus $T_l(\varepsilon)$ it is possible to obtain asymptotically representations.

The demonstrations of theorems use methods of the modern dynamical systems theory including the theory of invariant manifolds and the normal forms method.

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EXPANSION OF THE STRONGLY INTERACTING SUPERFLUID FERMI GAS: SYMMETRY AND SELF-SIMILAR REGIMES

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We consider an expansion of the strongly interacting superfluid Fermi gas in the vacuum in the so-called unitary regime when the chemical potential $\mu \propto \hbar^2 n^{2/3}/m$ where n is the density of the Bose-Einstein condensate of Cooper pairs of fermionic atoms. Such expansion can be described in the framework of the Gross-Pitaevskii equation (GPE) [1]. Because of the chemical potential dependence on the density $\sim n^{2/3}$ the GPE has additional symmetries resulting in existence of the virial theorem connected the mean size of the gas cloud and its Hamiltonian. It leads asymptotically at $t \to \infty$ to the ballistic expansion of the gas. We carefully study such asymptotics and reveal a perfect matching between the quasi-classical self-similar solution and the ballistic expansion of the non-interacting gas. This matching is governed by the virial theorem derived in [2] utilizing the Talanov transformation [3] which was first obtained for the stationary self-focusing of light in the media with cubic nonlinearity due to the Kerr effect. In the quasi-classical limit the equations of motion coincide with 3D hydrodynamics for the perfect gas with $\gamma = 5/3$. Their self-similar solution describes, on the background of the gas expansion, the angular deformations of the gas shape in the framework of the Ermakov–Ray–Reid type system.

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INTEGRABLE TURBULENCE IN OPTICAL FIBERS EXPERIMENTS

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Exactly integrable partial differential equations (PDEs) such as the Korteweg-de-Vries (KdV) or the one-dimensional nonlinear Schrdinger equation (1DNLSE) can be studied in the framework of the Inverse Scattering Transform (IST) also called nonlinear Fourier transform. Integrable PDEs exhibit an infinite hierarchy of invariants that prevent the development of standard Wave Turbulence and energy cascade. Despite the existence of the IST technique, there is no general theory describing of the propagation of random waves in integrable systems such as 1DNLSE. For this reason, Integrable Turbulence, which deals with random fields, has been recently introduced as a completely new chapter of turbulence theory by V.E. Zakharov, one of the creators both of the wave turbulence theory and of the IST [1].

Optical fibers are very favourable experimental platforms for the investigation of integrable turbulence. One of the fundamental problem of integrable turbulence is the understanding of the nonlinear stage of the noise driven Modulation Instability (MI). In this talk, we report recent optical experiments fiber experiments devoted to the investigation of the statistical and spectral properties of the long term evolution of a noisy unstable plane wave (condensate) [2]. Our experiments are very well described by the focusing one-dimensional nonlinear Schrodinger equation (1DNLSE) and provide the first observation of phenomena predicted by Agafontsev and Zakharov in 2015 [3].

Moreover, our data analysis reveal new understanding of integrable turbulence.

Despite the numerous studied devoted to the MI in optical fibers, it is only recently that the single-shot observation of MI spontaneously emerging from noise has been reported [4]. However, in these pioneering studies, the detection performed at the output of the fiber did not allow the observation of the dynamics along the propagation inside the fiber and the phase of the field could not be measured.

In this talk, we first present experiments performed in a recirculating fiber loop setup similar to the one reported in [5]. Our setup enables the first experimental observation of the space-time dynamics of the noise-driven MI in optical fibers [2]. Performing single-point statistical analysis of optical power recorded in the experiments, we observe decaying oscillations of the second-order moment together with the exponential distribution in the long term evolution, as predicted in [3]. Finally, we demonstrate experimentally and numerically that the autocorrelation of the optical power $g^{(2)}$ exhibits some unique oscillatory features typifying the nonlinear stage of the noise-driven modulation instability and of integrable turbulence.

In this talk, we also present completely new experiments allowing the measurement of the discrete eigenvalues of the IST spectra of integrable turbulence. The development of real-time measurement techniques for ultra-fast optical signals has attracted considerable interest over the last few years and has contributed to outstanding progresses in both applied and fundamental research. In particular, we have developped a new technique-heterodyne time lens-allowing the single shot measurement both of the phase and of the amplitude of random light [6]. Using this technique, we record the phase and amplitude of the noise-seeded MI of a plane wave in a 500-m-long optical fiber. The measured IST eigenvalues distribution at the input and output of the fiber are very similar, thus demonstrating the proximity to integrability of our optical fiber experiments. We discuss the influence of high order terms in the experiments. Finally, we show that our experiments provide a clear link between soliton gas and integrable turbulence.

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MODULATION INSTABILITY AND SOLITON DYNAMICS WITH TIME-DEPENDENT NONLINEARITIES

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By operating the photorefractive crystals in the non-instantaneous region, we report nonlinear resonance in modulation instability (MI) by experimental measurements and theoretical analyses based on a nonlinear non-instantaneous Schrdinger equation. With a periodic modulation in the external bias voltage, which acts equivalently as a modulation in the nonlinear strength, a resonance spectrum is disclosed with an enhancement in the visibility of MI at resonant frequency. A nonlinear manifold of a damped oscillator is demonstrated through spontaneous optical pattern formations, which is believed to be manifested also in other branches with nonlinear physics. Moreover, the non-trivial soliton formation and related dynamics will also be revealed with a periodic modulation in the nonlinear coefficient.

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INTEGRABILITY OF FULLY NONLINEAR KELVIN-HELMHOLTZ INSTABILITY DYNAMICS FOR COUNTERFLOW OF SUPERFLUID AND NORMAL COMPONENTS OF HELIUM

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Kelvin-Helmholtz instability (KHI) is perhaps the most important hydrodynamic instability which commonly occurs either at the interface between two fluids moving with different velocities or in the presence of the tangential velocity jump/shear flow in the same fluid. Recently KHI attracted significant experimental and theoretical attention in superfluids. KHI was studied either for interface between different phases of ^{3}He [1, 2], which has many similarities with KHI in classical fluids, or KHI from relative motion of components of ^{4}He [3, 4, 5] which has no classical analog thus we refer to it as quantum KHI. We focus on quantum KHI of the free surface in the presence of counterflow of superfluid and normal components of ^{4}He. A prin-

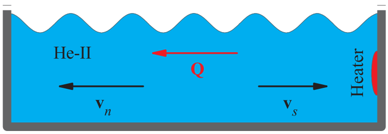

Fig. 4: A schematic of counterflow in superfluid ^{4}He. Heater results in the flux of heat $\mathbf{Q}$ which is carried by the normal fluid component with velocity $\mathbf{v}_n$ while superfluid component moves in the opposite direction with the velocity $\mathbf{v}_s$. Both components coexist in the same volume of fluid and share the same free surface.

ciple difference here from KHI of classical fluids is that relative fluid motion in quantum KHI occurs not from different sides of interface but from the same side of the He-II free surface with fluids components coexisting in the same volume which is purely quantum effect. A counterflow is achieved in experiment by the action of a stationary heat flow within the liquid in the direction tangent to the free surface as shown in Fig. 4. Linear analysis of both classical KHI and quantum KHI results in the exponential growth of surface perturbations. As these initial perturbations reach amplitudes comparable with their wavelength, nonlinear effects must be considered. For general nonlinear case each of two fluid components satisfy Euler equations with the kinematic and dynamic boundary conditions. We use a key property of quantum KHI that both fluid components share the same volume. It allows to use the conformal map from the area occupied by the fluid to the lower complex half-plane [7] We found, through the change of variables from two-fluid description into the effective single-fluid description and switching to the harmonically conjugated potential [8], that the linearly unstable branch of solution develops into fully nonlinear solution through the exact reduction to the Laplace growth equation (LGE). LGE has the infinite number of exact solutions often involving logarithms as well as it is integrable in a sense of the existence of infinite number of integrals of motion and relation to the dispersionless limit of the integrable Toda hierarchy [9]. LGE generically produces a finite time cusp singularity $y + 3/2k \propto |x|^{2/3}$ at the fee surface. We found that the surface tension regularizes the cusp and results in Crapper-like solutions.We notice that a single fluid case with nonzero gravity and surface tension turns more complicated with the infinite set of moving poles solutions found in Refs. [10, 11] which are however unavoidably coupled with the emerging moving branch points in the upper half-plane generally producing the infinite number of sheets of the Riemann surface [12]. Residues of poles are the constants of motion. These constants commute with each other in the sense of underlying non-canonical Hamiltonian dynamics [10, 13]. It suggests that the existence of these extra constants of motion provides an argument in support of the conjecture of complete Hamiltonian integrability of 2D free surface hydrodynamics [14].

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PERIODIC WAVES IN A SYSTEM OF WEAKLY COUPLED KDV-TYPE EQUATION

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A system of weakly coupled KdV-type equations is considered [1], [2]. Such systems appear in describing of a strong interaction between two different internal solitary wave modes in three-layer fluid. It means that phase speeds of these modes are nearly coincided [3]. In the work a branching of parametric families of periodic solutions is investigated. Lyapunov – Schmidt method is used. It allows us to reduce the original problem to a system of bifurcation equation, which analysis leads to sufficient condition of solutions orbits branching [4].

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GRAVITATIONAL REDSHIFT OF LIGHT SIGNALS IN A BREATHER-LIKE DARK MATTER HALO

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Although dark matter does not interact directly with ordinary matter, its oscillations lead to the oscillations of the gravitational field, which can be observed. We study the effect of these oscillations on light signals. A simple formula is obtained for the frequency shift of a light signal from a source located in the center of a spherically symmetric oscillating dark matter halo. Using this formula, we calculate the gravitational redshift of the light signal from the center of the breather-like lump in the scalar dark matter model with a logarithmic potential. The solution of the Einstein-Klein-Gordon system, describing this self-gravitating lump, was obtained in [1]. Interestingly, at some distances from the source and for some amplitudes of the breather, we found a blueshift instead of a redshift.

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EVOLVING SPECTRA OF HYDRODYNAMIC AND WAVE TURBULENCE

S. V. Nazarenko

I will talk about non-stationary self-similar evolution of spectra in strong hydrodynamic and weak wave turbulence. Special attention will be made to inverse cascades leading to Bose-Einstein condensation in optical and cold-atom turbulent systems.

DESTRUCTION OF ADIABATIC INVARIANCE IN DYNAMICS OF CHARGED PARTICLES NEAR MAGNETIC FIELD NULL LINE

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Study of charged particles motion in electromagnetic fields is a rich source of problems, models, and new phenomena for nonlinear dynamics. We consider a planar motion of a charged particle in stationary strong magnetic and electric fields. The magnetic field is perpendicular to the plane of particle motion and vanishes on some line - the magnetic field null line. Dynamics far from the null line is approximately described by the guiding centre theory, which is based on conservation of an adiabatic invariant – the magnetic moment of the particle. This approximation ceases to work near the magnetic field null line. This leads to remarkable phenomena which are new both for nonlinear dynamics in general and for the theory of charged particles motion. Particle dynamics switch between the slow guiding centre motion and the fast traverse along a segment of the magnetic field null line. Upon each null line traverse, the magnetic moment changes in a random fashion, causing the particle choose a new trajectory of the guiding centre motion. This leads to a stationary distribution of the magnetic moment, depending only on the particle's total energy. The jumps in the adiabatic invariant are described by Painlevé II equation.

PATTERN UNIVERSES

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Patterns turn up all over the place in both nature and laboratories. They arise from phase transitions in which systems far from equilibrium are stressed beyond certain thresholds. Some symmetries are broken; some remain. In large aspect ratio systems, because of remaining symmetries, patterns tend to consist of a mosaic of patches of some preferred planform (stripes, hexagons), often with a preferred scale, whose orientations are chosen by local biases. These patches meet and meld at defects, line and point in two dimensions, and as loops, targets and scroll waves in three dimensions. In gradient systems, patterns eliminate defects and slowly coarsen towards a state of minimum energy but this process takes a long time. In non-gradient systems, the defects can remain forever. Therefore, pattern defects are an integral part of the emerging state. They are universal and ubiquitous. They contain both topological charge and energy. In this talk, I explore parallels between these pattern universes and current thinking in cosmology. It is remarkable that the defects arising in systems starting out with the simplest of symmetries can exhibit objects with fractional charges of integer multiples of 1,1/2 and 1/3. They also contain energy and we show that this energy can lead to an additional force which results in the rotation velocities of stars in galaxies flattening to constant values (rather than decreasing), a phenomenon that has been widely observed and for which the purely mystical notion of dark matter was invented to explain.

GENERALIZED HERMITE POLYNOMIALS AND MONODROMY-FREE POTENTIALS

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We consider a class of monodromy-free Schrödinger operators with rational potentials constituted by generalized Hermite polynomials. These polynomials defined as Wronskians of classic Hermite polynomials appear in a number of mathematical physics problems as well as in the theory of random matrices and 1D SUSY quantum mechanics. Being quadratic at infinity, those potentials demonstrate localized oscillatory behavior near origin. We derive explicit condition of non-singularity of corresponding potentials and estimate a localization range with respect to indices of polynomials and distribution of their zeros in the complex plane. It turns out that 1D SUSY quantum non-singular potentials come as dressing of harmonic oscillator by polynomial Heisenberg algebra ladder operators. To this end, all generalized Hermite polynomials are produced by appropriate periodic closure of this algebra which leads to rational solutions of Painlevé IV equation. We discuss the structure of discrete spectrum of Schrödinger operators and its link to monodromy-free condition.

INTEGRABLE SYSTEMS AND COMBINATORICS

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I will talk about the wonderful facts: special solutions of integrable systems generate various combinatorial objects.

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DELAYED NONLINEAR RESPONSE OF METALLIC STRUCTURES AFTER LASER IRRADIATION

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The talk is devoted to the nonlinear effects in metallic structures (like gratings and nanoparticle arrays) irradiated by intense femtosecond laser pulses. Possible mechanisms of delayed low-frequency response and specific non-quadratic nonlinear regimes are considered. Also, the review of the recent experimental results on subpicosecond terahertz (THz) pulses generation from metals is presented. In general, special attention is paid to the role of plasmonic resonances at optical frequency in the enhancement of low-frequency nonlinear currents.

We discuss a microscopic theory of THz radiation generation on metal gratings under the action of femtosecond laser pulses. In contrast to previous models, only low-frequency currents inside the metal are considered without accounting for electron emission and acceleration. The presented analytical model is based on plasmon-enhanced thermal effects and explains the resonant character of delayed low-frequency response giving an adequate estimation for the overall pulse energy. We have performed numerical study of the discussed mechanism of THz currents and fields generation on the basis of hydrodynamical description of free electrons in metal. Modeling results show a resonant character of the low-frequency response when the laser pulse incidence angle is varied. Numerical modeling also allows to interpret such specific experimental features like low conversion efficiency when the grating depth is too large or existence of the optimal thickness if a structured foil is used.

It is also demonstrated theoretically that ultrafast heating of metal nanoparticles by the laser pulse should lead to the generation of coherent THz radiation during the heat redistribution process. It is shown that after the femtosecond laser pulse action the time-dependent gradient of the electronic temperature induces low-frequency particle polarization with the characteristic timescale of about fractions of picosecond. In the case of the directed metallic pattern, the THz pulse waveform can be controlled by changing geometry of a particle. The proposed mechanism can became the basis for interpretation of the recent experiments on the THz generation from metallic nanoparticles and nanostructures.

SET THEORETICAL SOLUTIONS OF THE YANG-BAXTER EQUATION AND THEIR ASSOCIATED INTEGRABLE MAPS

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We show that set theoretical solutions of the Yang-Baxter equation can be used to construct a hierarchy of integrable maps in higher dimensions. We give several examples of such integrable maps in dimension 3 and we explore their integrability and dynamical properties. We also generalise this construction by considering solutions of the set theoretical entwining Yang-Baxter equation and we present new integrable maps.

THREE-DIMENSIONAL REDUCTIONS OF THREE-DIMENSIONAL INTEGRABLE SYSTEMS

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In this talk we consider the Manakov–Santini three-dimensional quasilinear system. We describe infinitely many its three-dimensional hydrodynamic reductions, which are also three-dimensional integrable quasilinear systems. For these systems we discuss the method of two-dimensional hydrodynamic reductions.

TSUNAMI WAVES: NONLINEAR PHYSICS AND GEOPHYSICAL APPLICATION

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Analysis of historic information about tsunamis in the World shown that in average tsunami occurred once per month, and catastrophic event – once per year. Only in 2018, two damaged tsunamis occurred in Indonesia. The first one occurred on Sulawesi Island after strong earthquake on 28 September: wave height is 11 m and more 2000 died. The second one is related with eruption of Anak Krakatau volcano on 22 December: wave height is up to 7 m. and about 1000 killed. It is important to mention tsunami of landslide origin in Bureya River (Russia) in December: wave height is 40 m. Also, meteotsunami with 1.5 m occurred in Spain on 16 July. This list demonstrates the variety of mechanisms of tsunami generation. Modern mathematical models of tsunami description will be described. The role of nonlinearity and dispersion will be discussed. Various geophysical applications of these models will be presented.

FREQUENCY COMB SOLUTIONS FOR QUADRATIC NONLINEARITY

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Generation of frequency combs in $\chi^{(3)}$ optical micro-resonators attracted a great interest during the last decade [1]. Working comb devices employ dissipative 1D temporal solitons with a double balance: nonlinearity-dispersion and gain-losses. Realization of solitonic comb regimes in $\chi^{(2)}$ optical micro-resonators is nowadays a big challenge. While quadratic solitons caused cascading SH-OPO processes have been extensively explored [2], they are relevant to the conservative case. Addition of an external pump and linear losses for the resonator modes dramatically changes the situation. In contrast to the $\chi^{(3)}$ case, the difference in the group velocities for the first and second harmonics is generally the main factor determining nonlinear regimes above the OPO threshold, whereas the dispersion broadening is of minor importance. We show for the first time (analytically and numerically) that dissipative $\chi^{(2)}$ solitons and localized periodic states, where the difference in the group velocities is compensated by the quadratic nonlinearity, are possible. The model considered includes the main ingredients of the SH-OPO processes in $\chi^{(2)}$ micro-resonators: quasi-phase-matching of the first and second harmonics [3], a frequency detuning between a monochromatic external pump and a high-Q resonator mode, and realistic decay and material constants.

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HYDRODYNAMIC 2D TURBULENCE AND BEAM SELF-CLEANING IN MULTIMODE OPTICAL FIBERS

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Spatiotemporal light beam dynamics in multimode optical fibers (MMFs) has emerged in recent years as fertile research domain in nonlinear optics. Intriguing spatiotemporal wave propagation phenomena such as multimode optical solitons and parametric instabilities leading to ultra-wideband sideband series, although predicted quite a long time ago, have only been experimentally observed in MMFs in the last few years.

It is well known that linear wave propagation in MMFs is affected by random mode coupling, which leads to highly irregular speckled intensity patterns at the fiber output, even when the fiber is excited with a high quality, diffraction limited input beam. Recent experiments have surprisingly discovered that the intensity dependent contribution to the refractive index, or Kerr effect, has the capacity to counteract such random mode coupling in a graded index (GRIN) MMF, leading to the formation of a highly robust nonlinear beam. The self-cleaned beam at the fiber output has a size that is close to the fundamental mode of the MMF, and sits on a background of higher-order modes (HOMs). Typically, spatial self-cleaning is observed in several meters of GRIN MMF at threshold power levels of the order of few kWs, orders of magnitude lower than the value for catastrophic self-focusing. Moreover, self-cleaning is most easily observed in a quasi-continuous wave (CW) propagation regime (i.e., by using sub-nanosecond pulses), so that dispersive effects can be neglected. In such regime, nonlinear mode coupling is the sole mechanism responsible for the self-cleaning of the transverse spatial beam profile at the fiber output. So far, although different tentative explanations have been provided, the physical mechanism leading to Kerr beam cleaning remains an open issue.

Our theoretical study and experiments indicate that Kerr beam self-cleaning results from parametric mode mixing instabilities, that generate a number of nonlinearly interacting modes with randomized phases – optical wave turbulence, followed by a direct and inverse cascade towards high mode numbers and condensation into the fundamental mode, respectively. This optical self-organization effect is analogue to wave condensation that is well-known in hydrodynamic 2D turbulence.

We experimentally demonstrate that beam self-cleaning in multimode optical fibers is a process that conserves the average mode number, in spite of the dramatic nonlinear change and self-organization into the fundamental mode of the output intensity pattern. The process of energy flow into the fundamental and HOMs at the expense of modes with intermediate wave numbers originates from a parametric instability. These results provide yet another demonstration of the parallelisms between hydrodynamic and optical turbulence, and of the universality of mechanisms for spatial pattern generation in different physical settings.

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INDUCED DYNAMICS

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To explain the title I remind some aspects of dynamics of singularities of solutions of the integrable equations (sine-Gordon, KdV). This dynamics (or dynamics of zeros of the “tau-functions”) being highly nontrivial by itself is induced by trivial (linear) evolution of soliton parameters. This suggests to consider some other examples of systems defined by zeros of functions which parameters evolve linearly. In this way we get systems of finite number of degrees of freedom that besides being Hamiltonian and Liouville integrable ones, describe by means of the classical mechanics such quantum effects as bound states and creation/annihilation of particles.

COMPLICATED MODES OF TWO DELAY-COUPLED OSCILLATORS WITH A RELAY NONLINEARITY

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We consider a system of two relay type differential-difference equations with delay in the coupling between oscillators:

$$\begin{aligned} \dot u_1 &= \big(\lambda F\big(u_1(t-1)\big) + b\, G\big(u_2(t-h)\big)\ln(u_*/u_1)\big)u_1, \\ \dot u_2 &= \big(\lambda F\big(u_2(t-1)\big) + b\, G\big(u_1(t-h)\big)\ln(u_*/u_2)\big)u_2. \end{aligned} \tag{1}$$

Here

$$F(u) \stackrel{\text{def}}{=} \begin{cases} 1, & 0 < u \le 1, \\ -a, & u > 1, \end{cases} \qquad G(u) \stackrel{\text{def}}{=} \begin{cases} 0, & 0 < u \le 1, \\ 1, & u > 1, \end{cases} \tag{2}$$

$a, b = \text{const} > 0$, $u_* = \exp(c\,\lambda)$, $c = \text{const} \in \mathbb{R}$, $h > 1$.

In this work, we develop an approach, firstly proposed in [1], to model chemical synapses. Our approach is based on a modified idea of fast threshold modulation. It allows a system of two synaptic coupled neurons to have two dynamical effects at the same time: multistability and bursting.

The fast threshold modulation phenomenon (FTM), which was originally described in [2,3], is a special way of coupling of dynamical systems. Its characteristic feature is that the right-hand sides of the corresponding differential equations experience jumps as some control variables cross critical values. Here the terms $b\,G(u_{j-1})u_j\ln(u_*/u_j)$ change their sign from plus to minus as the potentials u_j increase and cross the critical value u_*.

An important feature of the system (1) is the presence of an additional time delay $h > 1$ in the coupling between oscillators. The second important feature is that (1) is independent phenomenological model of two synaptic coupled neurons. The presented approach allows us to consider only relay system (1) which is given a some biological meaning. This avoids a laborious proof of the correspondence theorems which one has to prove if right parts of (1) are continuous and parameter λ is large .

After the passage to the new variables (see [4])

$$x_j = (1/\lambda)\ln u_j, \quad j = 1, 2 \tag{3}$$

system (1) can be represented in the form

$$\begin{aligned} \dot{x}_1 &= R\big(x_1(t-1)\big) + b\,(c - x_1)\,H\big(x_2(t-h)\big), \\ \dot{x}_2 &= R\big(x_2(t-1)\big) + b\,(c - x_2)\,H\big(x_1(t-h)\big), \end{aligned} \tag{4}$$

where

$$R(x) \stackrel{\text{def}}{=} \begin{cases} 1, & x \le 0, \\ -a, & x > 0, \end{cases} \qquad H(x) \stackrel{\text{def}}{=} \begin{cases} 0, & x \le 0, \\ 1, & x > 0. \end{cases} \tag{5}$$

Note that substitute (3) leads (2) to relay functions (5), in particular

$$F(\exp(\lambda x)) = R(x),\ G(\exp(\lambda x)) = H(x) \text{ as } \lambda > 0.$$

A main result is the following. There exists a, b, c, h, $\sigma > 0$ such that system (4) admits $2n-1$ periodic modes colon $(x_1^{(m)}(t), x_2^{(m)}(t))$ $(m = 1, \ldots, 2n-1)$. Here $x_1^{(m)}(t)$ and $x_2^{(m)}(t)$ are $T^{(m)}$-periodic functions which have $2n-m$ and m relatively short alternating segments of positivity and negativity which go after a long enough segment where the functions values are negative. By (3), the existence of such a solution of system (4) means that (1) has a regime wich components have exactly $2n-m$ and m asymptotically high spikes on a period after a refractive segment. So, for any natural n we find a mechanisms of occurrence of $(2n-1)$ stable relaxation periodic regimes. The components of the solutions have summary $2n$ spikes on a period. Thus, there are both *multistability* and *bursting-effect*.

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NOVEL COLLECTIVE EXCITATIONS AND UNIVERSAL BROADENING OF CYCLOTRON ABSORPTION IN DIRAC SEMIMETALS

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Studies of phase states with linear dispersion of the spectrum of low-energy electron excitations are reviewed [1]. Some basic properties of these states in the so-called Dirac materials are discussed in detail. Results of modern studies of symmetry-protected electron states [2] with non-trivial topology are reported. A combination of geometry-based approaches with methods and results of condensed matter physics enables one to clarify new universal features of topological insulators, Dirac semimetals, and Weyl semimetals. We analyzed possible electron transitions from the region of occupied states to the region of unoccupied states for the relativistic spectrum of massless Dirac electrons in a quantizing magnetic field. Collisionless Landau damping regions of longitudinal and transverse collective excitations propagating along the magnetic field in Dirac semimetals were found. The existence of novel left-hand polarized modes in the windows of the Landau damping regions is indicated. In particular, we described [3] universal broadening of cyclotron absorption in Dirac semimetals.

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BLOW UP IN A FREE BOUNDARY PROBLEM FOR NAVIER — STOKES EQUATION

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We consider a class of exact solutions for Navier-Stokes equations describing the motion in strip or layer, one or both plane boundaries of which are free while another boundary can be a solid wall (problem A and B, respectively). In problem B, sufficient conditions for global existence of solution or its blow up in a finite time are obtained. Problem A is an analog of L.V. Ovsiannikov solution (1967) to the Euler equation There are three regimes of motion: stabilization to the rest state, blow up in a finite time and intermediate self-similar regime, in which the layer or strip expand infinitely for infinite time. Both problems admit three equivalent formulations: in Euler, Lagrange and Crocco variables. Each of them is suitable for certain purposes. Thin structure of blowing up problem B solution in plane case was studied by V.A. Galaktionov and J.L. Vazquez (2000). We give the upper estimate of both problems solutions lifespan in terms of initial data. It is remarkable that viscosity cannot prevent solution collapse in problem A. Numerical simulation confirms inviscid asymptotic as the main term of solution near moment of its destruction.

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INTERMITTENT PARTICLE AND ENERGY TRANSPORT IN MAGNETICALLY CONFINED PLASMAS THE ROLE OF COHERENT STRUCTURES

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Turbulence is the dominating mechanism for transporting particles, energy and momentum in the edge and scrape-off-layer (SOL) of toroidal magnetically confines hot plasmas. The transport is strongly intermittent and involves large outbreaks of hot plasma. These bursts of plasmas are formed in the edge region of the plasma and propagate far into the SOL. They appear as localized - in the poloidal plane perpendicular to the magnetic field - coherent structures of excess plasma pressure and are referred to as plasma blobs. They are stretched along the magnetic field as cigar shaped filamentary structures. These blobs have significant influence on the transport of plasma out of the confined region and the density and temperature profiles in the edge region as well as on the power load at plasma facing components, see, e.g., Ref. 1.

We present here a discussion of recent results on the formation and dynamics of the coherent blob structures, their influence on the transport and dynamics in the SOL plasma and their influence on the power load on plasma facing components. The investigations are based on theoretical models of blob structures and numerical simulations of the emerging blob structures in turbulence.

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TOPOLOGICAL REACTIONS AND TRANSFORMATIONS OF 3D-TANGLE LASER SOLITONS

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The interest in 3D-topological optical solitons is caused both by the richness and uncommonness of their properties, and by their increased stability and potential for informational applications. Indeed, in a homogeneous nonlinear dissipative medium of sufficiently large dimensions, a large number of such solitons can be formed, and when coding information with topological indices, essential is their preservation even with significant distortions or variation in the system parameters.

In [1–3], a family of localized topological structures of light in a homogeneous laser medium with fast saturable absorption were studied. The governing equation is the generalized complex Ginzburg–Landau equation for a slowly varying envelope of the electric field strength. With the use of previously studied 2D laser solitons, a family of 3D tangle-solitons was numerically found and investigated. They are characterized by their skeleton — a set of unclosed and closed vortex lines on which the field vanishes and the energy flows form vortices. When a control parameter (here the linear laser gain) slowly varies and crosses the boundary of the stability region, a number of topological reactions and soliton transformations occur. When the gain restores its initial value, the soliton topology can also returns to the initial or change. This report analyzes the corresponding dynamics.

The first example is provided by apple solitons with the same topologe (one closed and one unclosed vortex lines), but different symmetry, that are stable in overlapping parameter regions. After the cycle of increase and decrease of the gain, the final soliton is the same as the initial one. Correspondingly, in the absence of soliton topology change, the hysteresis is reversible.

For more complex initial solitons, after the loss of stability, a number of topological reactions occur; the elementary reactions are reconnection of vortex lines and separation of closed loops from a parent vortex line. An example is a "Hopf+" soliton with two linked closed vortex lines (the Hopf link) and three unclosed lines. Its stability region overlaps with those for a number of other topological solitons. Now after the hysteresis cycle, the final is "apple" soliton. For this soliton, as compared with the inital "Hopf +" soliton, the field energy decreases, the laser medium energy increases, and soliton topology is simplified.

The results determines the range of parameter variations in which soliton topological structure preserves and information coded by the topological indices are stored.

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STABILITY OF SOLITONS AND STABLE COLLAPSE IN NLS, HARTREE AND ZAKHAROV–KUZNETSOV EQUATIONS

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We first discuss stable blow-up dynamics in the nonlinear Schrodinger and Hartree equations. The last one is a Schrödinger-type equation with a nonlocal, convolution-type potential:

$$iu_t + \Delta u + \left(|x|^{-(N-\gamma)} * |u|^p\right)|u|^{p-2}u = 0,$$

$x \in \mathbb{R}^N$, describing the dynamics with long-range attractive interaction; in a context of boson stars it appears in the theory for stellar collapse.

Understanding a stable self-similar collapse in the case of the NLS equation goes back to works in 80's of Zakharov and others. In particular, for the 2d cubic NLS a generic collapse exhibits the *log-log* blow-up rate with the ground state profile (a positive vanishing at infinity solution of $Q = \Delta Q + Q^p$). Collapse in the same NLS equation in 3d shows a pure square root blowup rate $\|\nabla u(t)\|_{L^2} \sim (T-t)^{-\frac{1}{2}}$ (as it is L^2-supercritical), but the profile (coming from a slightly different profile equation) is only known from simulations; it is still a challenge to find the correct profile analytically, and in fact, it's been shown that there exist various multi-bump profile solutions. We show that all of this holds in higher dimensions in both L^2-critical and supercritical settings, including the case where the equation is energy-supercritical.

For the Hartree equation the question of self-similar collapse has been completely open; we show that in the L^2-critical setting the collapse behaves similarly to the self-similar collapse in the standard NLS, with *log-log* dynamics. In the supercritical Hartree equations, it also has similar features as in the L^2-supercritical NLS, though in several cases one has to be careful since the decay of the profile becomes polynomial. Nevertheless, our results suggest that the form of the nonlinearity in the NLS-type equations does not affect the formation and dynamics of the self-similar collapse.

Next, we consider Zakharov–Kuznetsov equation $u_t + \partial_{x_1}(\Delta_{\mathbb{R}^N} u + u^p) = 0$, $x = (x_1, ..., x_N)$, and investigate asymptotic stability of solitons or existence of collapse depending on the dimension N and generalized nonlinearity p. This equation can be considered as a higher-dimensional version of the (one dimensional) Korteweg-de Vries (KdV) equation, for which a question about existence of blow-up solutions for higher power nonlinearities (gKdV) has posed lots of challenges and far from being answered. One of the main obstacles is that unlike other dispersive models such as the NLS, the gKdV equation does not have a suitable virial quantity which is the key to showing existence of collapse. Partially, the question of existence and formation of singularities intertwines with the soliton stability (or the instability, which may lead to a blowup). Only at the dawn of this century analytical works started to appear on the existence of finite-time blow-up solutions for the quintic (critical) gKdV equation.

We consider, first, the 2d cubic (critical) Zakharov–Kuznetsov equation and investigate the existence of blow-up. We positively answer this question in this two dimensional version of Zakharov–Kuznetsov equation and also obtain the asymptotic stability in the subcritical setting (note that any Zakharov–Kuznetsov equation is non-integrable). We then consider the original 3d quadratic Zakharov–Kuznetsov equation and show the *asymptotic* stability of solitons. The main ingredients include the Liouville-type theorem and understanding the spectral properties for the virial-type quantities for the linearized adjoint equation. (Parts of this work is joint with Anudeep Kumar Arora (FIU, USA), Luiz Farah (UFMG, Brazil), Justin Holmer (Brown, USA), Kai Yang(FIU, USA) and Yanxiang Zhao (GWU, USA).)

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LONG-LIVED QUANTUM VORTEX KNOTS AND LINKS IN A TRAPPED BOSE-EINSTEIN CONDENSATE

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The dynamics of the simplest torus vortex knots, “unknots,” and links in an atomic Bose condensate at zero temperature in an anisotropic harmonic trap has been simulated numerically within the three-dimensional Gross-Pitaevskii equation. It has been found that such quasistationary rotating vortex structures exist for a very long time in wide ranges of the parameters of the system. This new result is qualitatively consistent with a previous prediction based on a simplified one-dimensional model approximately describing the motion of knotted vortex filaments.

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ENSEMBLE DYNAMICS AND THE EMERGENCE OF CORRELATIONS IN WAVE TURBULENCE IN ONE AND TWO DIMENSIONS

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We investigate statistical properties of wave turbulence by monitoring the dynamics of ensembles of trajectories. The system under investigation [1] is a simplified model for surface gravity waves in one and two dimensions with a square-root dispersion and a four-wave interaction term. The simulations of decaying turbulence [4] confirm the Kolmogorov-Zakharov spectral power distribution of wave turbulence theory [2, 3]. Fourth order correlations are computed as ensemble-averages of numerically computed trajectories. The shape, scaling and time-evolution of the correlations agrees with the predictions by wave turbulence theory. Instabilities that can lead to coherent processes that supersede wave turbulence are discussed [5, 6].

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I. THE THEORY OF ROGUE WAVES AT WORK IN A NONLINEAR OPTICS EXPERIMENT, AND II. THE ROGUE WAVE RECURRENCE IN NLS TYPE EQUATIONS

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In the first part of the talk we discuss the recent observation of Fermi-Pasta-Ulam rogue wave recurrences in a photorefractive crystal, in the light of the NLS rogue wave theory recently developed in collaboration with P. Grinevich [1,2,3,4,5], a joint work with Grinevich and with the nonlinear optics group of C. Conti and E. DelRe, Univ. of Roma “La Sapienza” [6]. In the second part of the talk we discuss the rogue wave recurrence in other NLS type equations like the Ablowitz-Ladik [7] and the PT-symmetric NLS [8] equations, emphasizing the novel and richer aspects of this recurrence with respect to the NLS case (joint work with my PhD student F. Coppini).

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ADIABATIC LIMIT IN GINZBURG-LANDAU AND SEIBERG–WITTEN EQUATIONS

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We consider the $(2 + 1)$-dimensional Higgs model governed by Ginzburg–Landau Lagrangian. The moduli space of its static solutions, called otherwise vortices, were described by Taubes. A description of slowly moving dynamical solutions may be given in terms of adiabatic limit. In this limit dynamical Ginzburg–Landau equations reduce to adiabatic equations which coincide with Euler equation for geodesics on the moduli space of vortices with respect to the Riemannian kinetic metric determined by the kinetic energy of the model.

A similar adiabatic limit procedure can be used to describe approximately solutions of the Seiberg–Witten equations on 4-dimensional symplectic manifolds. In this case geodesics of kinetic metric are replaced by pseudoholomorphic curves while solutions of Seiberg–Witten equations reduce to families of vortices defined in the normal planes to the limiting pseudoholomorphic curve. These families should satisfy a nonlinear $\bar{\partial}$-equation which can be considered as a complex analogue of adiabatic equation. Respectively, arising pseudoholomorphic curves may be treated as complex analogues of adiabatic geodesics in $(2+1)$-dimensional case. In this sense Seiberg–Witten model may be considered as a $(2+2)$-dimensional analogue of the $(2+1)$-dimensional Ginzburg–Landau model.

NOVEL ESSENTIALLY 2-D EVOLUTION EQUATIONS AND COLLAPSES IN BOUNDARY LAYERS

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High Reynolds number boundary layers are ubiquitous in nature and in engineering context. Some important boundary layers are 3-d, i.e. the streamwise and spanwise velocities are comparable. Often, the velocity shear in the boundary layer coexists with density stratification. Advancing understanding of the physical mechanisms of laminar-turbulent transition of boundary layers is a challenge of huge fundamental and practical significance. Here we consider a nonlinear mechanism of bypass transition due to collapses and show its relevance for a variety of boundary layers including weakly stratified and essentially 3-d ones.

We examine evolution of weakly nonlinear broadband long-wave perturbations for a wide class of laminar boundary layers (including 3-d boundary layers) with and without weak density stratification. Both the 'rigid lid' and 'no slip' boundary conditions are considered. Employing a regular asymptotic procedure which utilises the smallness of the boundary layer thickness compared to the characteristic wavelength, smallness of nonlinearity, density stratification and viscous effects we derive a family of novel essentially 2-d evolution equations in the distinguished limit where nonlinearity is comparable to dispersion and viscous effects. These equations describe evolution of the amplitude of the streamwise velocity perturbations in two spatial dimensions and time, while the perturbation cross-boundary layer spatial dependence is prescribed (to leading order) by a solution of the Rayleigh boundary value problem. The equations are essentially 2-d generalisations of the Benjamin-Ono equation, the streamwise and spanwise scales of perturbations are comparable. The dispersion is described by pseudo-differential operators which reduce to the Benjamin-Ono dispersion for plane perturbations. In contrast to the studies of linear stability the perturbations are decaying in the linear limit. Depending on the assumed scaling the terms accounting for viscous effects may enter the evolution equations or drop into the next order. However, the role of viscous effects is crucial in both cases: the sufficiently large viscosity ensures linear viscous regime in the critical layer, which eliminates singularities in the asymptotic expansion occurring at larger Reynolds numbers.

The evolution equations are investigated analytically and numerically. It has been found that within the framework of these equations a wide class of initial conditions leads to collapse, i.e. development of a point singularity in finite time - a localised blow-up. The physical mechanism of the collapse can be interpreted as the self-focussing of 2-d solitary waves. Development of collapse from a localised initial perturbation was examined numerically by integrating the evolution equations. The work outlines the boundary of 'nonlinear stability' and its dependence on the parameters of the problem (stratification, Reynolds number, boundary layer profile). For certainty the boundary in parameter space was drawn for initial perturbations having Gaussian axisymmetric shape. All things equal, stratification and decrease of the Reynolds number lead to rise of the initial amplitude threshold for the collapse to happen. The results suggest an alternative bypass scenario of laminar-turbulent transition for a wide class of boundary layers.

SUPERFLUID TURBULENCE AT FINITE TEMPERATURES: ESTIMATION OF EFFECTIVE VISCOSITY

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We study freely decaying quantum turbulence by performing high resolution numerical simulations of the Gross-Pitaevskii equation (GPE) in the Taylor-Green geometry. We use resolutions ranging from 1024^3 to 4096^3 grid points. The energy spectrum confirms the presence of both a Kolmogorov scaling range for scales larger than the intervortex scale ℓ, and a second inertial range for scales smaller than ℓ. Vortex line visualizations show the existence of substructures formed by a myriad of small-scale knotted vortices. Next, we study finite temperature effects in decaying quantum turbulence by using the stochastic Ginzburg-Landau equation to generate thermal states, and then by evolving a combination of these thermal states with the Taylor-Green initial conditions under the GPE. We use finite temperature GPE simulations to extract mean free path by measuring the spectral broadening in the Bogoliubov dispersion relation that we obtain from the spatio-temporal spectra, and use it to quantify the effective viscosity as a function of the temperature. Finally, we perform low Reynolds number simulations of the Navier-Stokes equations, in order to compare the decay of high temperature quantum flows with their classical counterparts, and to further calibrate the estimations of the effective viscosity (based on the mean free path computations).

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LUGIATO-LEFEVER MODEL IN THE CONTEXT OF FREQUENCY COMB GENERATION

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I will discuss recent results that reveal new features of the frequency combs in ring resonators described by the Lugiato-Lefever model. These features are associated with a periodicity of the boundary conditions in a spatial coordinate and produce a host of new effects that are appearing in a recent wave of experimental results in fibre loop and microring resonators. First, I will describe how periodicity of the problem introduces coexisting nonlinear resonances and coexisting solitons. Second, I will introduce higher-order dispersion into the problem and describe effects associated with the interaction of the solitonic Cherenkov radiation with its parent soliton, after the radiation makes a complete round trip around the resonator. Third, I will present recent results on how the 2nd harmonic generation effects influence frequency comb generation.

SOLITARY SYNCHRONIZATION WAVES IN DISTRIBUTED OSCILLATORS POPULATIONS

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We demonstrate the existence of solitary waves of synchrony in one-dimensional arrays of oscillator populations with Laplacian coupling. Characterizing each community with its complex order parameter, we obtain lattice equations similar to those of the discrete nonlinear Schrödinger system. Close to full synchrony, we find solitary waves for the order parameter perturbatively, starting from the known phase compactons and kovatons; these solutions are extended numerically to the full domain of possible synchrony levels. For nonidentical oscillators, the existence of dissipative solitons is shown.

More Expanded Description

The dynamics of oscillator populations attracts a lot of interest across different fields of science and engineering. The paradigmatic and universal object of study is the Kuramoto model of globally coupled phase oscillators. It demonstrates a transition to synchrony. This effect is relevant to many systems (lasers, biocircuits, electronic and electro-chemical oscillators). In the case the oscillators are organised as an ordered medium or a lattice with a distant-dependent coupling, spatio-temporal patterns can be observed. The most popular are standing chimera states, where regions of synchrony and asynchrony coexist. While chimera patterns are typically stationary solutions, a size of which is a characteristic system size, a possibility of localised traveling waves of the complex order parameter in oscillatory media remains an open problem.

We report on solitary synchronization waves in a one-dimensional oscillatory medium with Laplacian coupling. Our first model is a lattice of subpopulations of phase oscillators. The main tool of our analysis is based on the Ott-Antonsen ansatz, allowing one to write closed equations for the complex order parameter. The resulting model resembles the nonlinear Schrodinger lattice; thus our model provides a link between the theory of synchronization and the theory of solitons. We find solitary waves in the lattice via a perturbation method, starting with compacton solutions for the fully synchronous case, and describe the full domain of existence of localized waves for different levels of synchrony. Furthermore, we show that for one-dimensional arrays with diversity of natural frequencies and with additional attractive coupling compensating this diversity, waves of synchrony exist as dissipative solitons. To show generality of synchrony waves, we also demonstrate them in an off-lattice model of a continuous oscillatory medium with an interaction defined through a convolution integral. The interaction kernel is of Laplacian type, the integral over which vanishes.

DARBOUX INTEGRABILITY OF DISCRETE TODA LATTICES

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It is widely known that two-dimensional Toda lattices corresponding to the Cartan matrices of simple Lie algebras are Darboux integrable, that is, they admit complete families of essentially independent integrals along both characteristics. These generalized Toda lattices are particular cases of the so-called exponential systems. Although some particular examples of discrete and semidiscrete analogs of Toda lattices were known before, systematic approach to discretization of exponential systems was proposed by Habibullin and collaborators in 2011.

We prove that in general, Habibullin's discretization of a Darboux integrable exponential system provides a Darboux integrable (semi)-discrete exponential system. We find explicit formulas for complete families of integrals along both characteristics for semidiscrete and purely discrete exponential systems corresponding to the Cartan matrices of the series A and C.

INTEGRABLE EVOLUTION SYSTEMS OF GEOMETRIC TYPE

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Integrable systems of the form

$$u_t^i = u_{xxx}^i + 3A_{jk}^i(\mathbf{u})\, u_x^j\, u_{xx}^k + B_{jkl}^i(\mathbf{u})\, u_x^j\, u_x^k u_x^l, \quad i, j, k, s = 1, \ldots, N, \quad (1)$$

where $\mathbf{u} = (u^1, \ldots, u^N)$, are considered. The invariant description of such system involves an affine connection Γ and a covariantly constant tensor of rank (1,3) satisfying special algebraic relations. If the torsion T of Γ equals zero then the we arrive at a symmetric space with a covariant deformation of triple Jordan system. Another class of examples is related with the Bol loops. In this case, $T \neq 0$ but the curvatute tensor vanishes.

The main integrable matrix examples of such systems are presented. All isotropic vector integrable systems of the form (1) are found in [1].

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MKdV EQUATIONS RELATED TO KAC-MOODY ALGEBRAS OF TYPE $D_4^{(k)}$

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We present one-parameter families of multi-component mKdV-type equations related to $D_4^{(1)}, D_4^{(2)}, D_4^{(3)}$, integrable via the inverse scattering method. The spectral properties of the relevant Lax operators are studied, and the inverse scattering problem is reduced to a multiplicative Reimann-Hilbert problem formulated on a set of $2h$ rays l_ν, where h is the Coxeter number for the corresponding algebra. The equations admit a Hamiltonian formulation and the corresponding Hamiltonians are given.

LUMP STRUCTURE AND DYNAMICS WITHIN THE KADOMTSEV–PETVIASHVILI EQUATION

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It is shown that the Kadomtsev–Petviashvili equation

$$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} - \frac{\partial^3 u}{\partial x^3}\right) + \frac{\partial^2 u}{\partial y^2} = 0, \tag{1}$$

describing weakly nonlinear wave processes in positive dispersion media (KP1 equation) has a reach family of multi-lump solutions (two-dimensional solitary waves) propagating at different angles to the main axis x. The structure of an obliquely propagating single skew lump is described in detail; it can be presented as.

$$u(\xi,\eta) = 24V_x \frac{3 + V_x\left[V_x\eta^2 - (\xi + V_y\eta)^2\right]}{\left\{3 + V_x\left[V_x\eta^2 + (\xi + V_y\eta)^2\right]\right\}^2}, \tag{2}$$

where $\xi = x + V_x t$, $\eta = y + V_y t$, and V_x and V_y are the components of lump velocity $\mathbf{V} = (V_x, V_y)$.

It is shown that skew multi-lump solutions can be also constructed in the analytic form, the results obtained are illustrated graphically. The dependence of stationary multi-lump structures on free parameters is discussed. The relevance of skew lumps to the real physical systems is discussed. The decay of KP lumps is considered for a few typical dissipations: the Rayleigh dissipation, Reynolds dissipation, Landau damping, Chezy bottom friction, viscous dissipation in a laminar boundary layer, and radiative losses caused by large-scale dispersion. It is shown that the straight-line motion of lumps is unstable under the influence of dissipation. Lump trajectories are calculated for two most typical models of dissipation, the Rayleigh and Reynolds dissipations. The interactions of multi-lumps are studied analytically and numerically. It is revealed a complex character of multi-lump interactions resembling cascade process in cosmic rays or in the interaction of elementary particles.

LYAPUNOV FUNCTIONS AND LONG-TERM ASYMPTOTICS FOR SOLUTIONS TO A COMPLEX ANALOGUE OF THE SECOND PAINLEVÉ EQUATION

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Consider the ordinary differential equation:

$$w''_{zz} + (2|w|^2 - z)w = 0, \quad z \in \mathbb{R}, \quad w \in \mathbb{C},$$

that is a complex analogue of the second Painlevé equation. Such an equation appears in the description of the transition layer in the problem of self-focusing [1]. The solutions with unbounded growing amplitude as $z \to +\infty$ and the solutions that tend to zero as $z \to -\infty$ are investigated. The goal is the construction of asymptotics for general solutions as $z \to \pm\infty$.

Note that the substitution $w(z) = R(z)\exp(i\Phi(z))$ in the equation leads to the following system: $R''_{zz} - R(\Phi'_z)^2 + (2R^2 - z)R = 0$, $R\Phi''_{zz} +$

$2R'_z\Phi'_z = 0$. It follows from the second equation that

$$\Phi(z) = c_2 + c_1 \int\limits_{z_0}^{z} \big(R(\zeta)\big)^{-2}\, d\zeta, \quad z_0 = \mathrm{const},$$

with arbitrary parameters c_1, c_2. The substitution of the expression for $\Phi(z)$ in the first equation leads to the ordinary differential equation for $R(z)$:

$$R'' + (2R^2 - z)R - c_1^2 R^{-3} = 0.$$

Note that asymptotics for particular solutions can be constructed in the form:

$$R_+ = \sqrt{\frac{z}{2}} + \sum_{k=2}^{\infty} a_k z^{-3k/2+1/2}, \quad z \to +\infty;$$

$$R_- = \sum_{k=0}^{\infty} b_k (-z)^{-3k/2-1/4}, \quad z \to -\infty.$$

Substituting these series in the equation and grouping the expressions of the same powers of z and $-z$ give the recurrence relations for determining the constant coefficients a_k and b_k. In particular, $a_2 = (1+16c_1^2)/(8\sqrt{2})$, $b_0 = |c_1|^{1/2}$, $b_1 = -|c_1|^{3/2}/2$.

In the first step, the Lyapunov stability of the particular solutions $R_+(z)$ and $R_-(z)$ as $z \to +\infty$ and $z \to -\infty$ is investigated. The stability ensures the existence of a family of solutions with a similar long-term behaviour. In the last step, asymptotics for such solutions is constructed by averaging method with a transition to Lyapunov function-angle variables.

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ELECTRICAL VARIATIES AND DISCRETE INTEGRABLE SYSTEMS

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The talk is based on the recent paper in co-authorship with V. Gorbounov [1]. We propose a new approach to studying electrical networks interpreting the Ohm law as the operator which solves certain Local Yang-Baxter equation. Using this operator and the medial graph of the electrical network we define a vertex integrable statistical model and its boundary partition function. As an evidence that this gives an equivalent description of electrical networks, we show that in the important case of an electrical network on the standard graph introduced in [2], the response matrix of an electrical network, its most important feature, and the boundary partition function of our statistical model, can be recovered from each other. We show that this approach provides a natural deformation for the Lusztig decomposition of the Borel unipotent subgroup [3]. This also pay attention on the essential action of the affine Weil group on some particular electrical varieties which produces some discrete integrable models deforming the Toda system analogs of [4].

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SOLITON AND BREATHER GENERATION IN THE GARDNER EQUATION

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The generation of the solitary and breather waves in the framework of integrable Gardner equation (extended version of the Korteweg–de Vries equation with both quadratic and cubic nonlinear terms) is discussed. Initial conditions are chosen in the form of long periodic sine wave or the gaussian-like cavity. The first type of disturbances model the internal tidal wave entering shallow waters and transforming then into an undular bore. Our numerical computations demonstrate the features of undular bore developing for different signs of the cubic nonlinear term. The solitons and breathers generated from initial pulse with vanishing tails like gaussian cavity may be found by applying the Zakharov - Shabat inverse problem technique. We choose specific shape of cavity when the Zakharov - Shabat scheme do not lead to real eigenvalues corresponded to solitons, only all eigenvalues are complex corresponded to the breathers. Then the Gardner equation solved numerically. It is shown that firstly, the solitary waves of opposite polarities at each cavity slope are generated, and breathers are formed only as the result of these solitary wave interactions.

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UNIDIRECTIONAL GRAVITY WAVES ON THE SURFACE OF A DEEP FLUID

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In his seminal paper from 1968, Vladimir Zakharov proved that the system of Euler equations describing the evolution of a perfect, incompressible, irrotational fluid with a free surface has a Hamiltonian structure in which the surface elevation $\xi(x,t)$ and the trace of the velocity potential at the surface $\psi(x,t)$, are the canonically conjugate variables [1]. For surface waves of small amplitude, the Hamiltonian admits a series expansion in integer powers of ξ. In the case of pure gravity waves there exists a symplectic coordinate transformation that eliminates all cubic terms from the transformed Hamiltonian [2]. Furthermore, due to the unexpected cancellation of the coefficients of all fourth order non-generic resonant terms in a two-dimensional flow of unlimited depth [3], the Hamiltonian can be put, at least formally, in a Birkhoff normal form up to order four [4]. This observation prompted Dyachenko and Zakharov to hypothesize that the Birkhoff normal form near an equilibrium point $(\xi,\psi)=0$ is integrable to every order [3]. Subsequently Craig and Worfolk showed that integrability already fails at degree five [4]. However, for two out of nine possible relative orientations of the wave vectors, the five-wave interaction amplitude is exactly zero [5]. This fact has remained unexplained to date. In our recent work, we have shown that the vanishing of the fifth-order interaction for particular orientations of the wave vectors is not accidental and deduced certain six-wave interactions to be zero [6]. The latter result would be quite difficult, if not impossible, to obtain by the known methods. Finally, for gravity waves propagating in one direction (either to the right or to the left) we obtain the following set of nonlinear partial differential equations:

$$\xi_t^{\pm} \pm i\psi_x^{\pm} = 0,$$
$$\psi_t^{\pm} + g\xi^{\pm} = \frac{(i\psi_x^{\pm})^2}{1 \mp i\xi_x^{\pm}}$$

where $f^{\pm} = P^{\pm} f$, $P^{\pm} = \frac{1}{2}(I \pm i\mathbb{H})$ are orthogonal self-adjoint projections onto positive and negative wavenumber components and $\mathbb{H}$

is the Hilbert transform. The derivation of these unidirectional evolution equations is done without truncating the Hamiltonian, and an exact solution is found in terms of Lambert’s W-function [7].

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V. E. ZAHAROV, E. A. KUZNETSOV AND KINETIC THEORY

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The hydrodynamic substitution, which is wellknown in the theory of the Vlasov equation [1-4], was developed in the papers of Zaharov and Kuznetsov in several directions. Hamilton formalism in kinetics was developed [1]. And it was used also for the case of Benny equations [2]. We shall discuss the development of these ideas [5-11]. In [5-6] E. Madelung and V. V. Kozlov outlined the simplest derivation of the Hamilton–Jacobi(HJ) equation, and the hydrodynamic substitution simply related this derivation to the Liouville equation [7, 8]. The hydrodynamic substitution also solves the interesting geometric problem of how a surface of any dimension subject to an arbitrary system of nonlinear ordinary differential equations moves in Euler coordinates (in Lagrangian coordinates, the answer is obvious). This has created prerequisites for generalizing the HJ method to the non Hamiltonian situation. The H-theorem is proved for generalized equations of chemical kinetics, and important physical examples of such generalizations are considered: a discrete model of the quantum kinetic equations (the Uehling–Uhlenbeck equations) and a quantum Markov process (a quantum random walk). The time means are shown to coincide with the Boltzmann extremes for these equations and for the Liouville equation [9]. This give possibility to prove existence of analogues of action-angles variables in nonhamiltonian situation. Applications to problem of chemical kinetic [10] and to forming of elements [11] in the Universe are proposed.

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FROM SOLITONS AND COLLAPSES TO HIERARCHY OF ATTRACTORS

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The real world is non-integrable and multidimensional, therefore we do need attractors to understand and describe the world. The world is knowable due to attractors. Almost all trajectories to attractors cannot be described analytically. People see attractors intuitively and it is the reason why most nouns in all dictionaries are names of attractors.The paradigm of attractors is supported by computers, which visualize what Poincare and Arnold saw a long time ago. Basically, computer codes should respect invariants and attractors will be visible to those who are learned to see.

Attractors are constructed from invariants (aka conserved quantities or integrals of motion), laconic and reliable rails of physics and life. Strong invariants are momentum, angular momentum, energy, electric charge, Poincare invariants, phase volume (Liouville theorem). Weak invariants slowly change with time but are useful. Examples are adiabatic invariants, specific entropy, vorticity, number of atom and molecules, wave action, the mass of Earth, number of dollars in circulation and number of traded shares, passengers on board etc. Solitons and solitary waves fascinate scientists, while most of them do not understand why these very special solutions of wave equations are so important. A solution of a nonlinear wave equation includes arbitrary functions. Even if we limit ourselves by stationary solutions, there are an infinite number of solutions of which soliton is just a particular case and should not be observable. Solitons are observable because they appeared in simulations and physicists do see simulations. Solitons are attractors of non-integrable wave systems because solitons are maximum or minimum of energy under conservation of invariants like wave action or momentum.

The soliton attractor explains rogue waves because the waves dispersion is modified by surface non-uniform currents, especially in the presence of islands.

Turbulent attractors in tokamak are known experimentally as canonical profiles of pressure and were explained theoretically by magnetic invariants. Particle pinch attractor of tokamaks is the most enigmatic.

Peaking of temperature and pressure at the center is natural because the center is heated and the boundary is cooled. Peaking of particles is a strong paradox because the source of particles is at the boundary.The attractor is a plateau on distribution function of angular magnetic momentum eA/c. It is known as a Turbulent EquiPartition (TEP). There is a simple reason for the hypothesis: poloidal coordinate is not invariant, there is a significant poloidal torque force due to trapped and near trapped particles. Therefore the plasma is frozen in the poloidal magnetic field only and the remaining Lagrangian invariant L is simple, fluid particles are attached to the poloidal magnetic field. The attractor predicted several phenomena, including Internal Transport Barriers.

ANALYTIC THEORY OF WIND-DRIVEN SEA

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Self-sustained analytic theory of wind-driven sea is presented. It is shown that wave field can be separated into two ensembles: Hasselmann sea that consists of long waves with frequency $\omega < \omega_H$, $\omega_H \sim 4 - 5\omega_p$ (ω_p is frequency of spectral peak), and Phillips sea with shorter waves. In the Hasselmann sea that contains up to 95 % of wave energy, resonant nonlinear interaction dominates over generation of wave energy by wind. White-cap dissipation in the Hasselmann sea in negligibly small. The resonant interaction forms a flux of energy to the Phillips sea which plays a role of universal sink of energy. This theory is supported by massive numerical experiments and explains majority of experimental facts accumulated in physical oceanography.

FRONTS, PULSES AND WAVE TRAINS IN REACTION-DIFFUSION EQUATIONS WITH CROSS DIFFUSION. ANALYTICAL SOLUTIONS

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Reaction-diffusion fronts, pulses and wave trains are described analytically in a piecewise-linear approximation of the FitzHugh-Nagumo equations with linear cross diffusion terms, which correspond to a pursuit-evasion process. The dynamical regimes of wave propagation are studied, and the shape of the waves is explored in detail. The oscillations with exponential decay in the wave profile are found so that a wavy pattern forms. The specific feature in the behavior of the oscillatory waves — the coexistence of several waves with different profile shapes and propagation speeds for the same parameter values of the model.

NONLINEAR DYNAMICS OF THE FREE CHARGED SURFACE OF AN IDEAL FLUID; FORMATION OF BUBBLES

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A flat free surface of a dielectric liquid with a free surface charge is unstable in the external electric field. In experiments with liquid helium with the free surface charged by electrons [1], the situation was realized where surface charges fully screen the electric field above the fluid surface. For this case, the region occupied by the fluid coincides with the region where it is necessary to calculate the electric field distribution. This allows us to apply the method of conformal mappings [2] for the analysis of boundary evolution. In paper [3], using this method, a wide class of exact solutions describing the nonlinear dynamics of the boundary has been found in the absence of gravity forces and capillarity.

In the present work we consider the evolution of the charged fluid boundary taking into account capillary and gravitational forces. The governing equations are solved numerically using time dependent conformal transformation of the region occupied by the liquid into a half-plane.

It is shown that the electrohydrodynamic instability development leads to the formation of dimples (which later transform into bubbles) on the surface. Under certain conditions (depending on the value of the applied electric field and on the initial velocity distribution), a tendency to splitting of dimples/bubbles has been observed. This process can be described using exact solutions previously obtained by us for equilibrium configurations of the constant-pressure region with charged boundary in 2D geometry [4]. These solutions describe splitting this region into separate segments whose number is determined by the electric charge value. Thus, as a result of the development of the boundary instability, an electric charge accumulates in the bubbles causing their possible subsequent splitting according to scenarios depending on the charge value.

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FORMATION OF SINGULARITIES ON THE FREE SURFACE OF AN IDEAL FLUID IN THE ABSENCE OF EXTERNAL FORCES AND CAPILLARITY

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A convenient approach to the description of potential motion of an ideal incompressible fluid with a free surface in the plane two-dimensional geometry is based on the method of conformal mappings [1]. This transformation makes it possible to reduce the initial (2+1)-dimensional problem of the motion of the fluid to a (1+1)-dimensional problem of the motion of its surface. Nevertheless, the resulting nonlinear equations remain rather complex and efficient methods of constructing solutions for them have not yet been developed.

In paper [2], it was found that Euler equations for the motion of the fluid with the free surface can be directly reduced to the complex Hopf equation. This allows constructing a number of multiparametric exact local solutions describing various unsteady flows with the formation of singularities.

In the present work, we combine the approach proposed in [2] and the method of conformal mappings. This makes it possible to reveal a class of unsteady flows of the fluid with the free surface in the absence of external forces and capillarity for which the equations of motion in conformal variables become linear and are easily solved analytically. Using found exact local solutions, we analyze various scenarios of interaction of singularities of the complex velocity with the free surface, which leads to the formation of cusps, drops, and bubbles in a finite time. In particular, when a single root singularity approaches the surface, sharp holes are formed in the surface. Drops “are cut” from the main body of the fluid by a pair of root singularities. Bubbles appear at the collision of countercurrent flows of the fluid [3].

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