Conference track «19.14 Probability theory and mathematical statistics»

Complexity of Flows with Interactions and Stochastic Analysis

Academic supervisor – Богачёв Владимир Игоревич

Shi Yifan

Student (master)

National Research University Higher School of Economics, Faculty of Mathematics, Moscow, Россия

E-mail: yshi@edu.hse.ru

Let μ_0 be a probability measure on $\mathbb{R}^d$ and W is a $\mathbb{R}$-valued Wiener sheet.

Определение 1. [1] The stochastic differential equation with interaction has the following form

$$\begin{cases} \mathrm{d}x(u,t) = a(x(u,t),\mu_t,t)\mathrm{d}t + \int_{\mathbb{R}} b(x(u,t),\mu_t,t,q)W(\mathrm{d}t,\mathrm{d}q), \\ x(u,0) = u, u \in \mathbb{R}^d, \\ \mu_t = \mu_0 \circ x(\cdot,t)^{-1}, t \geq 0. \end{cases}$$

One example of such equation is the Helmholtz point vortex model.

Let $\mathcal{H}$ be a real separable infinite-dimensional Hilbert space and $h \in C^2(\mathbb{R}^d;\mathcal{H})$ satisfy $\big(h(u),h(v)\big)_{\mathcal{H}} = \Gamma(u-v)$, where Γ is a positive definite kernel. Define the $\mathcal{H}$-valued process

$$h_t := \int_{\mathbb{R}^d} h(y)\,\mu_t(\mathrm{d}y) = \int_{\mathbb{R}^d} h(x(u,t))\,\mu_0(\mathrm{d}u).$$

The space $\mathfrak{M}_n$ is the set of all probability measures on $\mathbb{R}^d$ with finite n_{th} moment.

Теорема 1. *Suppose that in equation (1),* $\exists C \geq 0 : \forall u_1,u_2 \in \mathbb{R}^d, \nu_1,\nu_2 \in \mathfrak{M}_m, t \geq 0 :$

$$\|a(u_1,\nu_1,t) - a(u_2,\nu_2,t)\| + \left(\int_{\mathbb{R}^d} \|b(u_1,\nu_1,t,q) - b(u_2,\nu_2,t,q)\|^2\right)^{\frac{1}{2}} \leq C\left(\|u_1-u_2\| + \gamma_m(\nu_1,\nu_2)\right)$$

where γ_m is Wasserstein distance of order m, and the functions a and b are continuous with respect to all variables. Then $\{h_t\}_{t\geq 0}$ is a Markov process in $\mathcal{H}$. Its infinitesimal generator $\mathcal{L}$ is given as follows. Take an orthonormal basis $\{e_k\}_{k\geq 1}$ of $L^2(\mathbb{R}^d)$. For any $\Phi \in C_b^2(\mathcal{H})$,

$$(\mathcal{L}\Phi)(h) = D\Phi(h)\left[\int_{\mathbb{R}^d} Dh(y)\,a(y,\mu,t)\,\mu(dy)\right] + \frac{1}{2}\sum_{k=1}^{\infty} D^2\Phi(h)\big(\Sigma_k(h),\Sigma_k(h)\big)$$

where $\Sigma_k(h)$ is defined in

$$\Sigma_k(h) := \int_{\mathbb{R}^d} Dh(y)\left[\int_{\mathbb{R}^d} b(x,\mu,t,q)\,e_k(q)\,dq\right]\mu(dy), \qquad \mu = \mu_0 \circ h^{-1}.$$

Asymptotic properties of h are also studied.

Источники и литература

1) A. A. Dorogovtsev, Measure-valued Processes and Stochastic Flows. De Gruyter, 2023.