

Conditional expectations, group actions, and Hilbert C^* -modules

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We study conditional expectations of finite index on C^* -algebras and their relations with Hilbert C^* -modules. Our results develop two interconnected directions.

First, we extend the notion of finite index, both algebraic and topological, from conditional expectations to A -valued inner products on Hilbert C^* -modules. Within this framework, we characterize structural properties of modules such as completeness, projectivity and freeness in terms of finiteness of the associated index. This provides an operator-algebraic criterion distinguishing trivial and nontrivial K -theory classes and clarifies the obstruction to freeness in terms of localized Wannier-type functions.

Second, we study conditional expectations arising from group actions on algebras of matrix-valued functions. More precisely, we consider actions of a group G on $C_0(X, M_n(\mathbb{C}))$, where X is a locally compact Hausdorff space. Assuming that the action is isometric, uniformly continuous and that the induced action on X has finite orbits of uniformly bounded cardinality, we prove that an averaging procedure defines a conditional expectation onto the invariant subalgebra. Moreover, this conditional expectation is shown to be of topologically finite index.

Our approach is inspired by the Jones index theory for subfactors [4] and by the theory of Hilbert C^* -modules [1,5]. The obtained results extend earlier work on conditional expectations arising from group actions [3,2] and provide a unified framework for studying finiteness phenomena for conditional expectations and Hilbert C^* -modules.

Источники и литература

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