Секция «Вещественный, комплексный и функциональный анализ»

ON BOUNDEDNESS OF THE GENERALIZED RIESZ POTENTIAL IN LOCAL MORREY TYPE SPACES

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We study the boundedness from one general local Morrey-type space to another one of the generalized Riesz potential

$$\left(I_{\rho(\cdot)}f\right)(x)=\int_{\mathbb{R}^n}\rho(|x-y|)f(y)dy,\ x\in\mathbb{R}^n,$$

under certain assumptions on the kernel ρ. Our aim is to generalize the known results for the case of the classical Riesz potential I_α, in which $\rho(t)=t^{\alpha-n}$, $t>0$, $0<\alpha<n$.

Let, for a Lebesgue measurable set $\Omega\subset\mathbb{R}^n$, $\mathfrak{M}(\Omega)$ denote the space of all functions $f:\Omega\longrightarrow\mathbb{C}$ Lebesgue measurable on Ω, and $\mathfrak{M}^+(\Omega)$ denote the subset of $\mathfrak{M}(\Omega)$ of all non-negative functions.

Определение 1. Let $0<\theta\leq\infty$ and let $w\in\mathfrak{M}^+\big((0,\infty)\big)$ be not equivalent to 0. We denote by $LM_{p\theta,w(\cdot)}$ the local Morrey-type spaces, the space of all functions $f\in\mathfrak{M}(\mathbb{R}^n)$ with finite quasi-norms

$$\|f\|_{LM_{p\theta,w(\cdot)}}\equiv\|f\|_{LM_{p\theta,w(\cdot)}}(\mathbb{R}^n)=\|w(r)\|f\|_{L_p(B(0,r))}\|_{L_\theta(0,\infty)}$$

where $B(0,r)$ is the open ball centered at the origin of radius $r>0$.

Определение 2. Let $0<\theta\leq\infty$.We denote by Ω_θ the set of all functions $w\in\mathfrak{M}^+((0,\infty))$ which are non-equivalent to 0 and such that $\|w\|_{L_\theta(t,\infty)}<\infty$ for some $t>0$.

We define two classes of kernels ρ, namely S_{n,p_1,p_2} and $\tilde{S}_{n,p_1,p_2}$.

Let H be the Hardy operator

$$(Hg)(t):=\int_0^t g(r)dr,\ 0<t<\infty,$$

and

$$\mathbb{A}=\left\{\varphi\in\mathfrak{M}^+((0,\infty);\downarrow):\lim_{t\to\infty}\varphi(t)=0\right\}.$$

Теорема 1. *Assume that* $0<\theta_1$, $\theta_2\leq\infty$, $w_1\in\Omega_{\theta_1}$, $w_2\in\Omega_{\theta_2}$.

1. *Let* $1<p_1<p_2<\infty$ *or* $1\leq p_1<\infty$ *and* $0<p_2\leq p_1$, *and* $\rho\in S_{n,p_1,p_2}$. *If the operator* H *is bounded from one weighted Lebesgue space* $L_{\theta_1,v_1(\cdot)}(0,\infty)$ *to another weighted Lebesgue space* $L_{\theta_2,v_2(\cdot)}(0,\infty)$ *on the cone* $\mathbb{A}$, *where*

$$v_1(r)=w_1\left(\mu^{(-1)}_{n,\rho,p_1}(r)\right)\left|\left(\mu^{(-1)}_{n,\rho,p_1}(r)\right)'\right|^{\frac{1}{\theta_1}},\ r>0,$$

$$v_2(r)=w_2\left(\mu^{(-1)}_{n,\rho,p_1}(r)\right)\left(\mu^{(-1)}_{n,\rho,p_1}(r)\right)^{\frac{n}{p_2}}|(\mu^{(-1)}_{n,\rho,p_1}(r))'|^{\frac{1}{\theta_2}},\ r>0,$$

$$\mu_{n,\rho,p_1}(r)=\frac{\int_r^\infty\rho(t)t^{\frac{n}{p_1'}-1}dt}{\int_1^\infty\rho(t)t^{\frac{n}{p_1'}-1}dt},\ r>0,$$

then the operator $I_{\rho(\cdot)}$ *is bounded from* $LM_{p_1\theta_1,w_1(\cdot)}$ *to* $LM_{p_2\theta_2,w_2(\cdot)}$.

2. *Let* $p_1=1$, $0<p_2<\infty$ *and* $\rho\in\tilde{S}_{n,1,p_2}$. *Then the operator* $I_{\rho(\cdot)}$ *is bounded from* $LM_{1\theta_1,w_1(\cdot)}$ *to* $LM_{p_2\theta_2,w_2(\cdot)}$ *if and only if the operator* H *is bounded from* $L_{\theta_1,v_1(\cdot)}(0,\infty)$ *to* $L_{\theta_2,v_2(\cdot)}(0,\infty)$ *on the cone* $\mathbb{A}$.

Necessary and sufficient conditions ensuring the boundedness of the Hardy operator H on the cone $\mathbb{A}$, are known for any measurable functions v_1 and v_2 and any $0 < \theta_1, \theta_2 \leq \infty$. Application of these results and Theorem 1 allows us to obtain sufficient conditions for boundedness of the operator $I_{\rho(.)}$ from $LM_{p_1,\theta_1 w_1(.)}$ to $LM_{p_2,\theta_2 w_2(.)}$ for $p_1 > 1$ and necessary and sufficient conditions for $p_1 = 1$.

References

1) V.I. Burenkov, A. Gogatishvili, V.S. Guliyev, R.Ch. Mustafayev, Boundedness of the Riesz potential in local Morrey-type spaces, Potential Analysis, 35 (2011), no. 1, 67-87.