

ON MINIMIZATION OF STRICTLY DETERMINISTIC TIMED FINITE STATE MACHINES

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Finite State Machines (FSMs) are widely used as formal models for solving numerous tasks in software engineering, VLSI design, development of telecommunication systems, etc. To describe the behavior of real-time systems FSM model can be extended by adding to it a clock — a continuous time parameter simulated by a real variable. In Timed FSM (TFSM) input and output signals have time stamps, and transitions are marked by output delays. An output delay indicates how much time it is required to process an input signal and generate an output response (see [1], [2]). Minimization problem for TFSMs is of prime importance since it arises in many applications similar to the same problem for FSMs. This problem was studied in some details in [2] for the basic variant TFSM model. In our paper we consider a more advanced variant of TFSMs; in this real-time model the order in which output signals are generated is determined not only by the order in which the input signals are consumed, but also by the delay required for their processing. When two runs of a TFSM perform the same sequence of transitions the output signals may follow in different order depending on the time of receipt of the input signals. A TFSM is called strictly deterministic if every output sequence is uniquely determined by a sequence of fired transitions. We begin our study of minimization problem for the new model of TFSM with considering the problem of checking the strict determinism property.

Let $\mathcal{A}$ and $\mathcal{B}$ be a finite input and output alphabets, and let $\mathbb{R}^+$ be the set of non-negative reals. G is the set of input timed guards. Each guard in G is an interval $g = \langle u, v \rangle$ where $\langle \in \{ (, [\}$ and $\rangle \in \{),] \}$, $u, v \in \mathbb{R}^+$ and $0 < u < v$. A sequence of pairs $(x_1, t_1), (x_2, t_2), \dots, (x_n, t_n)$ where $x_i \in \mathcal{A}$ ($x_i \in \mathcal{B}$) for all $i \in [1..n]$; $t_1, t_2, \dots, t_n$ is increased sequence and $t_i \in \mathbb{R}^+$ are called *Input (Output) sequence*.

A TFSM with timed guards is a tuple $M = (I, S, O, h_S, s_0)$ where I and O are input and output alphabets, S is the finite non-empty set of states, $h_S \subseteq (S \times I \times O \times S \times G \times \mathbb{R}^+)$ is the transition relation. G is the set of input timed guards. Each guard in G is an interval $g = \langle u, v \rangle$

where $\langle \in \{(\cdot, [\cdot]$ and $\rangle \in \cdot), \cdot] \}$, $u, v \in \mathbb{R}^+$ and $u < v$.

TFSM is a transition system $M = (S, s_0, \rightarrow)$ where S is finite non-empty set of states, $s_0 \in S$ is the initial state, and $\rightarrow \subseteq S \times \mathcal{A} \times S \times \mathcal{B} \times G \times \mathbb{R}^+$ is a transition relation. A transition $(s, a, s', b, \langle u, v \rangle, d)$ means that TFSM M being in state s accepts an input i at time $t \in \langle u, v \rangle$ after TFSM M entered state s ; M produces an output o after d is expired [2]. Sequence of transitions $(s_0, a_1, s_1, b_1, \langle u_1, v_1 \rangle, d_1), \dots, (s_{n-1}, a_n, s_n, b_n, \langle u_n, v_n \rangle, d_n) \quad (*)$

converts an input sequence $\alpha = (a_1, t_1), (a_2, t_2), \dots, (a_n, t_n)$ to an output sequence $\beta = (b_{i_1}, \tau_1), (b_{i_2}, \tau_2), \dots, (b_{i_n}, \tau_n)$, if

- 1) $t_i - t_{i-1}$ holds for all $i, 1 \leq i \leq n$, assuming that $t_0 = 0$ where $t_0 = 0$
- 2) β is the permutation of $\gamma = (b_1, t_1 + d_1), (b_2, t_2 + d_2), \dots, (b_n, t_n + d_n)$ such that the sequence of second components increases.

An output sequence $R(\beta) = b_{i_1} b_{i_2} \dots b_{i_n}$ is called *reaction* of TFSM M to the input sequence α . A sequence of transition $(*)$ is called *strictly deterministic* if for all input sequences α_1 and α_2 it follows that $R(\beta_1) = R(\beta_2)$ where β_1 and β_2 are the output sequences for α_1 and α_2 . A TFSM M is called *strictly deterministic* if all its initial runs are strictly deterministic.

At the first stage we formulated and proved necessary and sufficient condition for strictly deterministic property of an individual sequence of transition.

Теорема 1. *The sequence of transition $(*)$ is strictly deterministic if and only if for all k, ℓ such that $1 \leq k < \ell \leq n$ holds at least one of the two inequalities: $d_k - d_\ell < \sum_{i=k+1}^{\ell} u_i$ or $d_k - d_\ell > \sum_{i=k+1}^{\ell} v_i$.*

Our next step will be to establish the necessary and sufficient conditions for the strict determinacy of TFSMs and to develop minimization techniques for strictly deterministic TFSMs.

Литература

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